The Standard for Minimal Academic Preparedness in Mathematics to Enter a Job-Training Program

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Abstract

All of the papers in this session address aspects of a project undertaken by the National Assessment Governing Board (NAGB) to conduct a set of so-called judgmental standard-setting (JSS) studies for the 12th-Grade National Assessment of Educational Progress (NAEP). The studies, conducted from March to July 2011, constituted NAGB's first effort to set cut scores on NAEP to represent minimal academic preparedness for college course placement and for entry into job-training programs. They addressed the question of whether students required the same academic knowledge, skills, and abilities to be minimally prepared for "college and career." The college course studied was the first entry-level, credit-bearing, post-secondary course that a student would take, and the five job-training programs studied were automotive master technician; licensed practical nurse (LPN); pharmacy technician; computer support specialist; and heating, ventilation, and air conditioning (HVAC) technician. Details of the studies are given in the papers by Luz Bay (2012) and Susan Loomis (2012).

In this paper, I describe the process of developing *borderline preparedness descriptions in mathematics* for the college course placement and for each of the job-training programs. The descriptions were developed relative to the content of the assessment, including only academic knowledge and skills that were consistent with the assessment framework in mathematics. The goal was to reach agreement across panelists from different college and job-training programs on the minimal level of knowledge, skills, and abilities required for students to be eligible to enter the course or program.

After describing the process we followed, I present the minimal academic requirements in mathematics common to the programs as analyzed by the mathematics content facilitators for the studies. The analysis notes objectives of the assessment that were included in the borderline preparedness descriptions and evaluates the relationship of the minimal preparedness descriptions to the placement of the bookmarks representing minimal preparedness. I end with some remarks about strengths and weaknesses of the JSS studies.

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Overview of the Judgmental Standard-Setting Studies

The assessment of college and career preparedness in mathematics is high on the agenda of efforts to set national standards and assess their implementation. The judgmental standard-setting (JSS) studies we helped conduct were part of a larger effort by NAGB to learn whether the 12th-grade NAEP could reasonably be used as an instrument to measure such preparedness.

The method used to set the standards was a modified bookmark procedure. For each occupation, a four-day workshop was held in which two replicate panels of 10 or so instructors from a program met to take and score a NAEP mathematics exam, review the Grade 12 objectives in the 2009 mathematics framework for NAEP (NAGB, 2008), develop and refine descriptions of borderline preparedness in mathematics for entering their program, work through a set of NAEP mathematics items to identify the knowledge and abilities being assessed, examine a sequence of NAEP mathematics items ordered by difficulty, place a bookmark in the sequence at the location of the cut score, and by examining booklets showing student performance on the items and iterating the process, arrive at an agreed-on cut score on a pseudo-NAEP scale as well as an agreed-on borderline preparedness description. The same process was simultaneously used for preparedness in reading, and the mathematics of two occupations was addressed in each workshop. Each workshop had a so-called process facilitator for each panel and a so-called content facilitator for each pair of panels. The JSS staff conducted a pilot study with panelists who taught an introductory college course or taught in an automotive master technician program. Then we held three operational sessions for the six categories of panelists. Table 1 shows the organization of the sessions.

Table 1
JSS Sessions and Workshops

JSS Session	Workshops	Panels
Pilot Study	College Preparedness	Mathematics Panels A & B Reading Panels A & B
	Automotive Master Technician	Mathematics Panels A & B Reading Panels A & B
Operational Session 1	College Preparedness	Mathematics Panels A & B Reading Panels A & B
	Automotive Master Technician	Mathematics Panels A & B Reading Panels A & B
Operational Session 2	LPN	Mathematics Panels A & B Reading Panels A & B
	Pharmacy Technician	Mathematics Panels A & B Reading Panels A & B
Operational Session 3	Computer Support Specialist	Mathematics Panels A & B Reading Panels A & B
	HVAC Technician	*Mathematics Panels A & B* *Reading Panels A & B*

Note: Adapted from Table 1 in Loomis (2012, p. 5).

Developing Borderline Preparedness Descriptions

Before coming to a workshop, panel members participated in an orientation webinar in which they got an overview of the process to be used in the JSS studies and became acquainted with the NAEP framework they would be using. They were asked to complete a survey as to how the content of the NAEP assessment corresponded to the requirements for students to be prepared to take a course for credit in college or in a job-training program. The content facilitators used their survey responses to draft a borderline performance description (BPD) for them to discuss and refine. Each panel was expected to reach agreement on the knowledge, skills, and abilities that students should have to be prepared for placement in a credit-bearing college-level course or in a job-training program. They were asked to describe the performance that distinguished preparedness for the course from the need for remediation. Before each round of bookmarking (setting the cut score) for preparedness, they were given an opportunity to review and revise the BPD. That process was seen as critical for developing a common understanding of the BPD, which was to serve as the criterion for setting the cut score to represent preparedness.

Twelfth-Grade NAEP Mathematics

The 2009 mathematics framework for NAEP (NAGB, 2008) contains objectives that are designed to assess students' use of quantitative tools, their broad competence in mathematical reasoning, their knowledge of the mathematics required for postsecondary courses, and their ability to integrate and apply mathematics in diverse problem-solving contexts. If one considers the entire domain of mathematics, only part of it is assessed in NAEP, and the NAEP frameworks need to be understood as assessment frameworks and not curricular frameworks.

Table 2
Percentage Distribution of Items by Grade and Content Area

<table>
<tr><th>Content Area</th><th>Grade 4</th><th>Grade 8</th><th>Grade 12</th></tr>
<tr><td>Number properties and operations</td><td>40</td><td>20</td><td>10</td></tr>
<tr><td>Measurement</td><td>20</td><td>15</td><td rowspan="2">30</td></tr>
<tr><td>Geometry</td><td>15</td><td>20</td></tr>
<tr><td>Data analysis, statistics, and probability</td><td>10</td><td>15</td><td>25</td></tr>
<tr><td>Algebra</td><td>15</td><td>30</td><td>35</td></tr>
</table>

Note: Adapted from Exhibit 2 in NAGB (2008, p. 6).

The 12th-grade NAEP items are classified into five content areas: number properties and operations; measurement; geometry; data analysis, statistics, and probability; and algebra. NAEP is administered at Grades 4 and 8 as well as at Grade 12, and Table 2 shows how the how the content emphasis changes across the grades. The area of number properties and operations drops in emphasis at Grade 12, but that is somewhat misleading given that it is used in all the areas. The percentage of measurement and geometry items is relatively high because the two categories are combined for the purpose of item distribution although separated in the framework. Also, the categories are combined at Grade 12 because "the majority of measurement topics suitable for 12th-grade students are geometric in nature" (NAGB, 2008, p. 5). The emphasis on data analysis, statistics and probability was increased in this framework over previous frameworks in large part because the area is so important today.

The 12th-grade items are also classified according to the level of thinking they require, which is termed *mathematical complexity* and classified as low, moderate, or high. Complexity is used only when the assessment is developed, for assuring a reasonable balance in demand across items; it is not used in reporting and was not used in the judgmental standard-setting studies.

Each student takes only a small sample of the NAEP items at any grade. Items are organized into blocks, and within the block, they are distributed across content areas, complexity levels, difficulty levels, and item formats. At Grade 12 in 2009, there were 348 mathematics items organized into 22 blocks (National Center for Education Statistics [NCES], 2010, p. 31). Each student took two 25-minute blocks.

> Some questions incorporated the use of a calculator, ruler/protractor, or other manipulatives that were provided. Twelfth-graders were permitted to use their own scientific or graphing calculator or were provided with a scientific calculator to use on approximately one-third of the assessment. (p. 31)

Mathematics Achievement Levels

NAEP sets so-called achievement levels with cut scores for each of the subjects assessed. These cumulative levels are termed *Basic, Proficient,* and *Advanced*. The NAEP mathematics scale at Grade 12 goes from 0 to 300, and the cut scores denoting the low end of the score range for each level are 141 for Basic, 176 for Proficient, and 216 for Advanced. The descriptions of the levels are as follows:

> Twelfth-grade students performing at the *Basic* level should be able to solve mathematical problems that require the direct application of concepts and procedures in familiar mathematical and real-world settings. . . .
>
> Twelfth-grade students performing at the *Proficient* level should be able to recognize when particular concepts, procedures, and strategies are appropriate, and to select, integrate, and apply them to solve problems. They should also be able to test and validate geometric and algebraic conjectures using a variety of methods, including deductive reasoning and counterexamples. . . .
>
> Twelfth-grade students performing at the *Advanced* level should demonstrate in-depth knowledge of and be able to reason about mathematical concepts and procedures. They should be able to integrate this knowledge to solve nonroutine and challenging problems, provide mathematical justifications for their solutions, and make generalizations and provide mathematical justifications for those generalizations. These students should reflect on their reasoning, and they should understand the role of hypotheses, deductive reasoning, and conclusions in geometric proofs and algebraic arguments made by themselves and others. Students

> should also demonstrate this deep knowledge and level of awareness in solving problems, using appropriate mathematical language and notation. (NCES, 2010, p. 31; see also NAGB, 2008, p. 73 & NCES, 2006)

The panelists were shown these descriptions, which they discussed along with sample items from NAEP showing student performance so that they would have a better understanding of how students scoring at each level performed. The achievement level descriptions helped provide the panelists with a context in which to formulate similar descriptions for borderline performance.

Minimal Preparedness in Mathematics

Developing the BPDs and relating them to the 2009 mathematics framework turned out to be a difficult task for all the groups, although less difficult for the college preparedness panels than for the job-training program panels. A major part of the problem appeared to be the academic language used in the draft descriptions and the framework. Even when the content facilitators attempted to reduce the jargon and provide concrete examples, the panelists struggled to understand what was being said.

For those occupational groups that seemed to have a common set of expectations for their programs, such as the automotive master technicians and the LPNs, the task of formulating BPDs was easier than for the groups whose programs seemed less homogeneous, such as the computer support specialists and the pharmacy technicians. The latter two groups had much more difficulty reaching consensus (Wilson, Lindquist, & Kilpatrick, 2011).

For the panels in Operational Session 3 (see Table 1), the computer support specialists and HVAC technicians, two high school teachers had been added to each panel to support the BPD development process. That modification appeared to help because the high school teachers could explain to the other panelists some of the technical terms in the framework and unfamiliar mathematics in the items. Unless they had experience with a post-secondary training program for the occupation, however, the teachers tended to have little idea of what the program or job required in the way of mathematics and tended to respond in terms of what their 12th graders knew instead of what program entrants might need to know.

Not surprisingly, the college preparedness group had higher expectations regarding algebra than any of the vocational groups. Those expectations were partially influenced by the college preparedness group's assumption that for the borderline performance descriptions, algebra was the entry-level course. That also helps explain why geometry and data analysis, statistics, and probability got less emphasis from that group (Wilson et al., 2011).

Below I summarize, by content area, the differences in level of agreement across vocational groups as analyzed by Wilson et al. (2011). For each area, I give a narrative description, a table, and a sample item. A check mark in a cell of the table indicates that the concept or set of skills was included in the final BPD of the group; a blank space indicates that it was not included. Text in the cell indicates that the concept or skill was included but the group clarified, elaborated, or otherwise edited the text in the mathematics framework when they formulated their BPD.

Number Properties and Operations

The objectives in the content area of number properties and operations received the strongest endorsement of the panelists. All six groups expected that students should be able to understand and perform arithmetic operations on numbers, especially fractions, decimals, and percents (Table 3). They thought students should be able to use estimation strategies and verify the reasonableness of results. They differed on the representations of numbers that they considered important; for example, some groups valued scientific notation, exponents, or absolute value more than others did. All considered proportions or proportional reasoning important, and all expected students to know and be able to apply properties of numbers and the number system. The college preparedness group expected a formal and theoretical knowledge of number properties and operations, whereas the occupational groups expected practical applications of those properties.

Table 3
Inclusion of Number Properties and Operations Objectives by Vocational Group

Group	Operations, esp. fractions, decimals, percents	Estimation	Representations	Proportions	Properties
College prep	all real numbers	✓	interpret & compare; include exponents & radicals	✓	formal
Auto master tech	include large & small numbers	✓	✓	✓	practical
LPN	include large & small numbers	✓	✓	✓	practical
Pharmacy tech	include large & small numbers	✓	✓	✓	practical
Computer support	✓	✓	✓	✓	practical
HVAC tech	rationals & common irrationals, scientific notation	✓	include absolute value	✓	practical

Figure 1 shows one of the few released 12-grade items in the number properties and operations content area. It assesses estimation and is of medium difficulty (52% correct in 2009). Although all the groups included estimation in their BPD, some had difficulty accepting this type of item because of the uncertainty in the answer. They preferred to use estimation to verify results. Some panelists, particularly those in occupations concerned with correct dosages or accurate measurement of tolerances, needed to be convinced that they could endorse estimation without implying that they did not care about precision in answers.

Measurement

Although measurement objectives were also strongly endorsed by the panelists, there was a difference between the expectations of the college preparedness group and those for the occupations (Table 4). The college prep BPD addressed standard school-based measurement ideas, whereas in the occupations, practical measurement ideas were central. The job-training program panelists wanted students to have an operational understanding of measurement—use tools with precision and accuracy, understand units of measure, be able to convert units within and across systems, and solve realistic problems. All six groups expected incoming students to be able to solve problems involving rates.

Number and Operations, Grade 12, 2009
Estimate amount of time for problem in context (calculator available)

5. The manager of a company has to order new engines for its delivery trucks after the trucks have been driven 150,000 miles. One of the delivery trucks currently has 119,866 miles on it. This truck has the same delivery route each week and is driven an average of 40,000 miles each year. At this rate, the manager should expect this truck to reach 150,000 miles in approximately how many months?

   1. Less than 4 months
   2. Between 4 and 6 months
   3. Between 6 and 8 months
   4. Between 8 and 10 months
   5. More than 10 months

The correct answer is D.
52% correct (47% incorrect; 1% omitted)

Source: http://nces.ed.gov/nationsreportcard/itmrlsx/

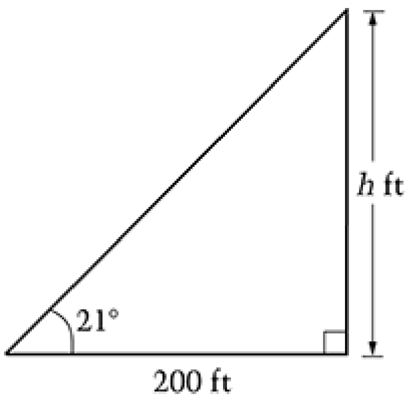
Figure 1. Released item on estimation.

Table 4
Inclusion of Measurement Objectives by Vocational Group

Group	Rate	Use tools	Understand units	Convert units	Realistic problems	Perimeter, area, volume, angles
College prep	✓					✓
Auto master tech	✓	✓	✓	✓	✓	✓
LPN	✓	✓	✓	✓	✓	✓
Pharmacy tech	✓	✓	✓	✓		
Computer support	✓	✓	✓	✓	✓	✓
HVAC tech	✓	✓	✓	✓	✓	✓

Figure 2 shows a released item in the measurement content area. It deals with measurement in triangles and is a hard item (30% correct in 2009). None of the vocational groups explicitly mentioned measurement in triangles in its BPD. The college preparedness group, however, did mention solving problems that involve measures of angles, and both the automotive master technicians and the HVAC technicians mentioned that they wanted incoming students to be familiar with indirect measures.

Measurement, Grade 12, 2009
Use trigonometry to find height of object



Note: Figure not drawn to scale.

11. On level ground from a distance of 200 feet, the angle of elevation to the top of a building is 21°, as shown in the figure above. What is the height h of the building, to the nearest foot?

1. 72
2. 77
3. 187
4. 201
5. 521

The correct answer is B.
30% correct (66% incorrect; 4% omitted)

Source: http://nces.ed.gov/nationsreportcard/itmrlsx/

Figure 2. Released item on indirect measurement.

Geometry

Geometry was the content area that the groups found the fewest objectives that were applicable to their program. There was no common geometry objective across all six groups (Table 5). The pharmacy technicians and computer support specialists included no geometry objectives. The LPN group's BPD statement asked for a minimal amount of familiarity with basic geometric terms and relationships. The college preparedness group wanted students to understand plane figures, apply the Pythagorean Theorem, and solve problems involving the coordinate plane. The automotive master technicians emphasized visualizing three-dimensional objects; they also mentioned analyzing relationships of lines. The HVAC technicians included the same geometry objectives that the automotive technicians did plus the objective of using properties and relationships of geometric figures, including the Pythagorean Theorem.

Table 5
Inclusion of Geometry Objectives by Vocational Group

Group	Plane figures	Coordinate plane	Visualize 3D	Properties of lines	Use properties & relationships of figures, incl. Pythagorean Theorem
College prep	✓	✓		✓	✓
Auto master tech			✓	✓	
LPN	minimal familiarity				
Pharmacy tech					
Computer support					
HVAC tech			✓	✓	✓

Figure 3 shows a released item in the geometry content area. It deals with coordinate geometry and is a hard item (8% complete credit and 5% partial credit in 2008). The item asks for a justification that the figure is a parallelogram. Any of several justifications—all depending on slopes, measures of line segments, or measures of angles—are acceptable. As noted above, the college preparedness group was the only one that explicitly mentioned the coordinate plane in its BPD.

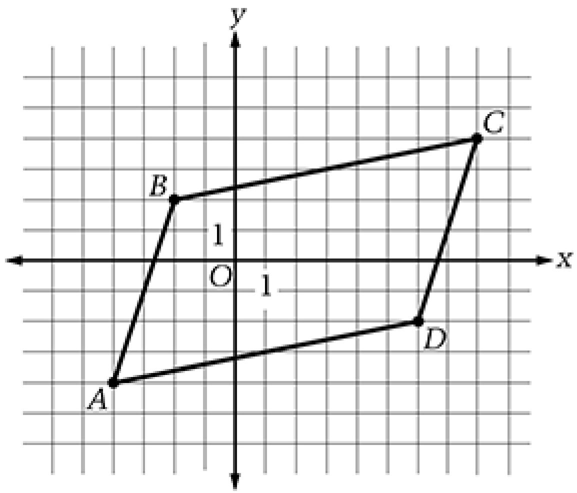
Data Analysis, Probability, and Statistics

All six groups wanted incoming students to be able to read and interpret graphs and tables (Table 6). The college preparedness group, LPNs, and computer support specialists supported the construction of graphs and tables, but the other three groups did not. All the groups valued basic measures of center and spread. All the groups except the college prep group were concerned with the ability to distinguish relevant from irrelevant information. Basic concepts of probability were mentioned by the college prep group, automotive master technicians, LPNs, and computer support group but not by the others.

Table 6
Inclusion of Data Analysis, Probability, and Statistics Objectives by Vocational Group

Group	Read and interpret graphs	Construct graphs	Distinguish relevant information	Basic probability
College prep	✓	✓		✓
Auto master tech	✓		✓	✓
LPN	✓	✓	✓	✓
Pharmacy tech	✓		✓	
Computer support	✓	✓	✓	✓
HVAC tech	✓		✓	

Geometry, Grade 12, 2009
Prove that given figure is a parallelogram



12. In the figure above, the vertices of ABCD are $A(-4,-4)$, $B(-2,2)$, $C(8,4)$, and $D(6,-2)$.

Give a mathematical justification that ABCD is a parallelogram.

Credit is given for any of several justifications: That both pairs of opposite sides are parallel, that both pairs of opposite sides are congruent, that one pair of opposite sides are both congruent and parallel, that both pairs of opposite interior angles are congruent, or that the diagonals bisect each other.

8% complete credit; 5% partial credit (64% unsatisfactory or incorrect justification; 20% omitted; 3% off task)

Source: http://nces.ed.gov/nationsreportcard/itmrlsx/

Figure 3. Released item on coordinate geometry.

Figure 4 shows a released item in the data analysis, probability, and statistics content area. It addresses an objective on experiments and samples—“recognize and describe a method to select a simple random sample”—and is of medium difficulty (60% correct in 2008). The pharmacy technician panelists said that minimally prepared students should understand the concept of bias, but they did not mention randomness. In their BPDs, none of the groups mentioned methods for drawing samples. The LPNs did say that incoming students should be able to recognize a random sample from a population, whereas the college preparedness group mentioned neither selection nor recognition of such samples.

Data Analysis, Probability, and Statistics, Grade 12, 2009
Identify appropriate method for selecting random sample

8. The principal of a high school would like to determine why there has been a large decline during the year in the number of students who buy food in the school's cafeteria. To do this, 25 students from the school will be surveyed. Which method would be the most appropriate for selecting the 25 students to participate in the survey?

1. Randomly select 25 students from the senior class.
2. Randomly select 25 students from those taking physics.
3. Randomly select 25 students from a list of all students at the school.
4. Randomly select 25 students from a list of students who eat in the cafeteria.
5. Give the survey to the first 25 students to arrive at school in the morning.

The correct answer is C.
60% correct (40% incorrect)

Source: http://nces.ed.gov/nationsreportcard/itmrlsx/

Figure 4. Released item on random selection.

Algebra
All six groups said that students should be able to write simple algebraic expressions, equations, or inequalities to represent a situation (Table 7). Students should also be able to evaluate algebraic expressions or formulas and solve equations and inequalities. Only the college preparedness group and the computer support specialists expected an ability to translate across representations, such as tabular, graphic, and symbolic representations. Only the college prep group had any expectations regarding functions.

Table 7
Inclusion of Algebra Objectives by Vocational Group

Group	Write expressions, equations, & inequalities	Evaluate expressions; solve equations	Translate across representations	Functions
College prep	✓	✓	✓	✓
Auto master tech	✓	✓		
LPN	✓	✓		
Pharmacy tech	✓	✓		
Computer support	✓	✓	✓	
HVAC tech	✓	✓		

Figure 5 shows a released item in the algebra content area. It deals with the solution of a pair of linear equations in two variables and is of medium difficulty (38% correct in 2008). None of the BPDs produced by the groups explicitly mentioned systems of equations.

Algebra, Grade 12, 2009
Solve a system of linear equations

14. What is the solution to the system of equations $\begin{cases} 3x - 2y = -7 \\ x + y = 11 \end{cases}$?

Answer: x = ____________________ y = ____________________

The answer is x = 3, y = 8.
38% both values correct; 6% one value correct (53% incorrect; 3% off task)

Source: http://nces.ed.gov/nationsreportcard/itmrlsx/

Figure 5. Released item on system of equations.

Although it is not explicitly noted in all the BPDs, the sense of every group's discussion supported the objective of minimally prepared students being able to solve multi-step application problems both within and across the five content areas.

Conclusion

There was a difference between the perspectives of the college preparedness group and the occupational groups in the type of mathematics they expected. The NAEP mathematics framework and items at Grade 12 assume a high school mathematics curriculum that has historically been based on college preparation and not vocational preparation. In other words, NAEP is more oriented toward pure mathematics than applied mathematics. Consequently, the job-training groups struggled to find the mathematics they valued in either the framework or the test items. Much of the mathematics at Grade 12 is well beyond what they would expect.

The areas of number properties and operations and of measurement were by far the most important content areas for every occupational group, yet they receive the least emphasis in the NAEP test. At Grade 12, the objectives for number properties and operations deal extensively with the real number system, which was not a priority for the panelists. Operations with fractions, decimals, and percents and their properties, which all the groups wanted incoming students to know, are treated in the Grade 8 objectives. In contrast to the college preparedness group, the job-training groups saw measurement as a necessity. Because the measurement objectives of the framework are more applications-oriented than the others, the job-training groups were better able to relate to those objectives than to the others. The college preparedness group was less willing than the other groups to choose objectives that were applications oriented.

Although any revision of the Grade 12 framework ought to deal more and better with applications of mathematics (so as to reflect recent curriculum changes), it will almost certainly remain inappropriate for setting a standard for minimal program preparedness. The Grade 8 objectives in the framework are much better suited to the mathematics needed for the occupations in this study.

As noted by Loomis (2012, p. 6), the replicate panels for the college preparedness, LPN, and computer support specialists groups set significantly different cut scores for minimal academic preparedness. Despite careful training, scripting, and coordinating, I observed that process facilitators took somewhat different approaches and consequently got somewhat different responses from their panels. Moreover, a combination of program heterogeneity and personality factors sometimes acted to move the panels in different directions.

Loomis (2012) also notes the high level at which the cut scores in mathematics appear to have been set. Part of the problem may be that despite repeated reminders from all the facilitators that the level of minimal preparation the panelists were to set was to apply to entrants to their programs as they exist today, many panelists seem to continue to think that by setting a cut score, they were sending a message as to what students entering their program should know. They were "raising the standard"—as it were.

In general, the NAEP items seem reasonably well suited to assessing students' achievement in mathematics at Grades 4, 8, and 12. The NAEP assessment does not, however, appear to be well suited as an instrument to determine whether a student is prepared to enter a specific course or occupational program. Any set of assessment items in mathematics that would indicate minimal preparedness for a course or program would require different criteria for developing and testing items, plus studies of how well those items predicted preparedness for course or program entry. Some occupations appear to have agreed-on criteria for successful program entry, which would make such studies easier to conduct. Other occupations, however, appear to have little or no consensus about entry criteria, so that would require substantial additional work before a useful instrument could be constructed and validated.

References

Bay, L. (2012, April 14). *Preparing job trainers to describe knowledge, skills, and abilities measured in an academic assessment*. Unpublished paper prepared for presentation at the annual meeting of the National Council on Measurement in Education, Vancouver, Canada.

Loomis, S. (2012, April 14). *A study of "irrelevant" items: Impact on bookmark placement and implications for college and career readiness*. Unpublished paper prepared for presentation at the annual meeting of the National Council on Measurement in Education, Vancouver, Canada.

National Assessment Governing Board. (2008). *Mathematics framework for the 2009 National Assessment of Educational Progress*. Washington, DC: Author.

National Center for Education Statistics. (2006). *The NAEP mathematics achievement levels by grade: Grade 12*. Retrieved from NCES website: http://nces.ed.gov/nationsreportcard/mathematics/achieveall.asp#grade12

National Center for Education Statistics. (2010). *The nation's report card: Grade 12 reading and mathematics 2009 national and pilot state results* (NCES 2011–455). Washington, DC: Institute of Education Sciences, U.S. Department of Education.

Wilson, L., Lindquist, M., & Kilpatrick, J. (2011, August 29). *A comparison of the six mathematics performance descriptions*. Unpublished manuscript.