Analogy in Learning and Reasoning

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http://www.psych.northwestern.edu/psych/people/faculty/gentner/
Why Study Analogy and Similarity?

- Core process in higher-order cognition
- A general learning mechanism by which complex knowledge can be acquired
- Unique to humans (or nearly so):

![Diagram]

- **Similarity**
  - *Species-general*
  - **Object match**
  - A
    - A
    - B

- **Analogy**
  - *Species-restricted*
  - **Relational match**
  - AA
    - BB
    - CD
SAMPLE ANALOGIES

The atom is like the solar system.

\[ 2 : 4 :: 4 : 8 \]
• An analogy conveys that partly identical relational structures hold between objects in different domains.

• Corresponding objects need not resemble each other.
How analogy leads to learning: New inferences

Clement & Gentner, 1991; Medin, Goldstone & Gentner, 1993; Spellman & Holyoak, 1989
How analogy leads to learning: Detecting differences

Alignable difference: different elements that each occupy the same role in the aligned structure:

e.g., “Motorcycles have two wheels, cars have four”
How analogy leads to learning:
Detection of differences

Analogical comparisons make alignable differences more salient

(Gentner & Markman, 1994; Markman & Gentner, 1993, 1996)

Example task: Find the wrongly placed bone
(Kurtz & Gentner, in prep)
How analogy leads to learning: Promoting relational abstractions

- Highlights common relational structure
- Supports abstraction of common structure

Analogy highlights common relational structure and fosters relational abstractions.

Gentner, 1983; Gentner & Markman, 1997; Gentner & Namy, 199; Gick & Holyoak, 1983

*e.g.*, A small force at a long distance from the fulcrum can Balance a large force at a small distance from the fulcrum.
Analogy highlights common relational structure and fosters relational abstractions.

Once understood, the relational abstraction can often be extended to more dissimilar pairs via Progressive alignment.

(Kotovsky & Gentner, 1986; Gentner, Anggoro & Klibanoff, in press)
Process Model (SME)

Structural alignment: initially blind, local-to-global process that ends structurally consistent

Three Stages

1. Local matches made in parallel; free-for-all

2. Structural consistency enforced: internally consistent mappings (kernels)

3. Kernels combined into maximal interpretation
   - Systematicity bias favors deeply interconnected matches
   - Structural evaluation computed
   - Candidate inferences projected

Inferences produced by pattern completion, not by hypothesis testing.
Structurally constrained

SME (Structure-mapping Engine) (Falkenhainer, Forbus & Gentner, 1989; Forbus, Gentner & Law, 1995)
Theories of Similarity

Similarity as Feature-Set Overlap (Tversky, 1977): Contrast Model

Representations: Objects as feature sets

\[ S(a,b) = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A) \]

\( \alpha > \beta \)

Add common features → More similar
Add distinctive features → Less similar
Theories of Similarity

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\[ \alpha > \beta \]

Add common features
\[ \rightarrow \text{More similar} \]

Add distinctive features
\[ \rightarrow \text{Less similar} \]

BUT - Sometimes,
Add distinctive features
\[ \rightarrow \text{MORE similar} \]
Commonalities, Differences, and Similarity

- **Hotel/Motel**
  - High Similarity

- **Traffic Light/Shopping Mall**
  - Low Similarity

List commonalities

- High Sim > Low Sim

List differences

- Contrast model (independent features) predicts:
  - Low Sim > High Sim

- Structure-mapping predicts:
  - Alignable differences
    - High Sim > Low Sim
Difference Listings

High Similarity Pair  Hotel/Motel  (5 AD, 0 NAD)
A motel is for driving to, a hotel is for vacations (AD)
At a motel, the room doors are outside, at a hotel they are inside (AD)
At a motel, there are only stairs, at a hotel there is an elevator (AD)
Hotels are in cities, motels are on the interstate (AD)
Hotels have pools more often than motels have them (AD)

Low Similarity Pair  Traffic Light/Shopping Mall  (1 AD, 4 NAD)
You can go inside a shopping mall, you can’t go inside a traffic light (NAD)
A shopping mall has stores, a traffic light doesn’t (NAD)
A shopping mall protects you from the weather, a traffic light doesn’t (NAD)
A traffic light can tell you when to go, a shopping mall doesn’t (NAD)

[PAUSE]
A shopping mall is on the ground, a traffic light is up in the air (AD)

Alignable difference:
Same dimension (predicate) with different values (arguments).

Non-alignable difference:
Information applying to one term but not the other.

Markman & Gentner, 1993
Result:

Easier to name differences for high-similar pairs than for low-similar pairs.

Given: 40 word pairs: 20 High Sim, 20 Low Sim

![Table]

<table>
<thead>
<tr>
<th>Similarity</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total differences:</td>
<td>5.88*</td>
<td>11.38</td>
</tr>
<tr>
<td>Alignable differences:</td>
<td>3.88*</td>
<td>9.09</td>
</tr>
<tr>
<td>Non-alignable differences:</td>
<td>2.00</td>
<td>2.28</td>
</tr>
</tbody>
</table>

Task: List one difference for as many pairs as possible in five minutes.

Results:
Computing similarity involves structural alignment

Proportion of Relational Responses

1MAP  Sim>1MAP
Comparison as structure-mapping

- Human comparison is a process of
  - Structural alignment
  - Followed by structure-sensitive projection of inferences

- Selection of which alignment to choose and of which inferences to project is via connected systems of relations

- Analogical comparison is a discovery mechanism for relational patterns
  - in scientific breakthroughs
    (Gentner, 2000; Holyoak & Thagard, 1995)
  - in learning and reasoning
    (Bassok, 1990; Bassok & Holyoak, 1989)
  - in conceptual development
    (Kotovsky & Gentner, 1996)
Good news, bad news, good news

1. Good news
   • Evidence for structure-mapping:
     • Comparison highlights connected relational structure

2. Bad news
   • *Inert knowledge* problem: failure to retrieve prior relational matches
     • Many missed opportunities for insight

3. Good news:
   • Two ways to promote relational encoding and transfer
     • One way to achieve later “rescue” of inert knowledge
Good News: Analogy gives rise to new knowledge
Invites relational abstraction:

- Wallcorp divested itself of Best Tires
- Likewise, Martha divorced George
  - Commonality: “They each got rid of something they no longer wanted”

- Invites new inferences that are structurally selective
  - Wallcorp divested itself of Best Tires and bought a more profitable tire company.
  - Likewise, Martha divorced George, and...
Good News: Analogy gives rise to new knowledge

Invites relational abstraction:

• Wallcorp divested itself of Best Tires
• Likewise, Martha divorced George
  • → Commonality: “They each *got rid of* something
    they no longer wanted”

• Invites new inferences *that are structurally selective*
  • Wallcorp divested itself of Best Tires
    and bought a more profitable tire company.
  
  • Likewise, Martha divorced George, and…
    
    • …*married a more advantageous man*

• *Not*: bought a tire company
Bad News – The *Inert Knowledge* Problem

People often fail to think of past analogous experiences that could help in current context

*Inert Knowledge*: Relationally similar items in LTM are often not retrieved

*Surface Intrusions*: Surface-similar items are often retrieved instead

Relational reminding and transfer often fails to occur

Proverbs: Continuous Reminding

- Relational Match (Analogy)
  - “You can’t judge a book by its cover”
  - “All that glitters is not gold”

- Object Match (Mere Appearance)
  - “You can lead a horse to water but you can’t make it drink”
  - “Don’t look a gift horse in the mouth”

Note: The proverbs used were unfamiliar
Proverbs: Relational Reminding and Judgments of Similarity

Continuous Reminding

<table>
<thead>
<tr>
<th></th>
<th>Identical</th>
<th>Mere Appearance</th>
<th>True Analogy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion Recalled</td>
<td>1.00</td>
<td>.90</td>
<td>.80</td>
</tr>
</tbody>
</table>

Disassociation

- Object similarity dominates in reminding
- Relational similarity dominates in judgments of similarity & inferential soundness.

Ross, Novick, Bassok, Holyoak & Koh; Gentner, Rattermann, & Forbus
Implications

Good news:
Comparison promotes
• alignment of common relational structure
• highlighting and abstraction of common relational system

Bad news:
Potentially fruitful prior exemplars are often not retrieved

Good news:
Comparison during encoding can make relational stucture more available for transfer
Analogical Encoding

- Standard analogical mapping from known base to new situation
  - align cases
  - project inferences

Analogical encoding over two new instances
Use comparison during learning
  - extract common system
  - store as relational abstraction
Learning negotiation strategies

- Highly motivated students
  - MBAs and graduates in business

- Challenging domain
  - New strategies
  - Must be flexibly recognized & applied
  - Often in ‘hot’ transfer situations

- Hard to learn
  - Even for experienced businessmen

NEGOTIATION STRATEGIES

Default: compromise on all issues
Better:
Trade-off: give each side more of what matters to them
Contingent contract: agreement scaled depending on future event

Gentner, Loewenstein & Thompson, 2003; Loewenstein, Thompson & Gentner (2003, 1999); Thompson, Gentner & Loewenstein, 2000
Comparison in learning negotiation strategies
Collaborators: Leigh Thompson (Kellogg) & Jeff Loewenstein (NU student)

- Highly motivated students (MBAs, business grads)
- Must flexibly recognize appropriate strategies, under “hot” conditions
- Strategies are notoriously hard to learn

Students study two analogous (surface-dissimilar) cases to prepare for a simulated negotiation

Study:

Separate Cases

Case 1
____________
____________
Case 2
____________
____________

Comparison

Case 1
____________
____________
Case 2
____________
____________

Simulated Negotiation

Read each case, write principle and/or give advice.

Compare the two cases and write the commonalities

Gentner, Loewenstein & Thompson, 2003; Loewenstein, Thompson & Gentner, 1999
Experiment 1: Proportion of negotiating pairs who use the strategy exemplified in the cases

Separate Cases: N=22 dyads

Prop. Forming Contingent Contracts: .23

Compare: N=22 dyads

Prop. Forming Contingent Contracts: .64 *

Proportion linking the initial cases:

Compare: .97

Separate Cases: 0

Loewenstein, Thompson & Gentner, 1999
Negotiation transfer performance across three studies: Proportion using strategy exemplified in the cases

- No Cases: N=42, Prop. Forming Contingent Contracts = 0.19
- Separate Cases: N=83, Prop. Forming Contingent Contracts = 0.24
- Compare: N=81, Prop. Forming Contingent Contracts = 0.58*
Implications

Comparison promotes

• alignment of common relational structure
• highlighting and abstraction of common relational system

So relational structure becomes

• more explicit
• less contextually embedded
• more portable
  • more likely to transfer to a new context

Analogy is a way to disembend knowledge
More Bad News

• Comparison induces a structural alignment, which promotes learning relational abstraction.

• So it offers a way of learning important conceptual knowledge—an alternative to innate belief systems in cognitive development

• BUT early in learning, novices and young children lack sufficient relational knowledge to succeed in aligning a pure analogy

Relational shift in similarity (Gentner, 1988)

e.g. “How is a cloud like a sponge?”
5-year-old: “Both are round and fluffy”
9-year-old: “Both hold water and later give it back”
Good News

Mundane similarity gives a small boost to relational salience.

Comparison – even of close literally similar examples – preferentially highlights relational commonalities

Example from children’s word learning
Collaborator: Laura Namy
Experiment 2

- 80 4-year-olds (M= 4;4)

“This is a *dax.* Can you show me which one of these is a *dax?*”

<table>
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<th>Group 2</th>
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<tbody>
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<td><img src="image1.png" alt="Image 1" /></td>
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</table>

Both groups prefer perceptual choice: .41 category.
• 80 4-year-olds \((M=4;4)\)

“This is a \textit{dax}. Can you show me which one of these is a \textit{dax}?”

These are both \textit{daxes}. Can you see why they’re both \textit{daxes}? …

Can you show me which one of these is a \textit{dax}?
Implications

Comparison – even mundane comparisons between closely similar examples – preferentially highlights common systems of relations

This provides a path for experiential learning of relational structure, even for young children

It also suggests that invisible learning of tiny structural generalizations may be occurring continually
Alternative Explanation

Gentner & Namy suggested that comparison between closely similar examples allowed children to focus on common systems of relations.

But children already know these categories, e.g., wheeled vehicles. Perhaps they are just accessing familiar categories more readily when they have two exemplars.
Test: can comparison promote novel relational abstractions?

Collaborator: Stella Christie
Teaching children novel spatial relations: Examples
Using comparison to teach novel spatial relations

Two conditions: Solo vs. Comparison

Two age groups:
3;10 (range 3;6-4;2; n=26)
4;8 (range 4.5-5.0; n=30)

8 unfamiliar relations -- each given a novel label

Competing object match
SOLO

“Which one of these two is also a Toma?”

Relational match

Object match
Results

Solo

![Graph showing results for Solo]

- For the age group 3;10 years, the value is 0.02*
- For the age group 4;8 years, the value is 0.25*
COMPARISON

“This is a Toma”

“And this is also a Toma”

“Can you see why they’re both Tomas?”

Relational match

Object match

“Which one of these two is also a Toma?”
- Both age groups significantly chose Object Match in the Solo condition
- Proportion of relational choices is significantly higher in Comparison than in Solo for both 3;10-year-olds ($d = 2.7$) and 4;8-year-olds ($d = .41$)
But are children really developing new insights through structural alignment?

Or are they just shifting among existing hypotheses:

- **H1.** “Blicket” = turtles
- **H2.** “Blicket” = smaller above bigger, otherwise identical

Must be H2

“Blicket” = smaller above bigger, otherwise identical
But are children really developing new insights through structural alignment?

Or are they just shifting among existing hypotheses:

H1. “Blicket” = *turtles*
H2. “Blicket” = *smaller above bigger, otherwise identical*

Test: make comparison difficult by showing examples sequentially

*It’s not about turtles – must be H2*
Study 2 - Same two exemplars presented sequentially

“This is a blicket”

“This is a blicket”

“Which one of these is a blicket?”

3;10 yrs: n= 14
4;8 yrs: n =14
Implications

• Seeing two exemplars separately does not promote relational insight.

• The comparison process itself contributes insight
Further Implications

Close alignment potentiates far alignment

Theoretical implication: Mundane literal similarity is processed with same structure-mapping process as analogy

Learning implication: Progressive alignment from close to far similarity can allow novice learners to make rapid progress

  children learning names for parts
  (Gentner, Loewenstein & Hung, 2007)

  Adults learning geoscience
  (Jee et al., 2008)
Analogy highlights common relational structure
So does literal similarity

- In analogy, two situations have the same *relational structure*
- If the corresponding objects are similar, alignment is easier

Gentner, 1983; Gentner & Markman, 1997
Some ways to foster comparison

High surface similarity

Physical juxtaposition

Direct invitation: “See how these are alike?”

Same word for both items

Symbolic juxtaposition
Spatial Mapping Task

Mapping Task

Experimenter’s Set

Child’s Set

Simple Objects

*Correct Match

Competing Object Match

- Rule: Same relative size and position
- 14 trials with feedback.
- 3-, 4- and 5-year-olds

Gentner & Rattermann (1991)
Difficult (Cross-mapped) Spatial Mapping Task

→ Object matches compete with relational rule

Rattermann & Gentner (1998, in prep.)

Experimenter’s Set

Child’s Set

Sparse O’s

Rich O’s

Baseline Performance

Proportion Relational Responses

0.2

0 0.4 0.6 0.8 1

chance

Rich Sparse

0.2

0.4

0.6

0.8

1
Learning relational language enables 3-year-olds to master the relational rule

Training

<table>
<thead>
<tr>
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</tr>
<tr>
<td>‘Baby’</td>
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Test: Difficult Spatial Mapping Task

<table>
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<th>C</th>
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**Training**

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**Results**

- 5 w/labels
- 4
- 3

**Test: Difficult Spatial Mapping Task**

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Proportion Relational Responses

- Rich
- Sparse

*chance
Summary of “Daddy/Mommy/Baby” Studies

• Children perform better on a difficult spatial mapping task when taught language that highlights the relational structure

• Children retained this understanding
  • when the label were withdrawn
  • and when retested 4-6 weeks later
Summary of “Daddy/Mommy/Baby” Studies

• Children perform better on a difficult spatial mapping task when taught language that highlights the relational structure

• Children retained this understanding
  • when the label were withdrawn
  • and when retested 4-6 weeks later

Similar results with another simple mapping task
(Loewenstein & Gentner, 2005)

3-year-olds perform much better if we use “top middle bottom” or “on in under”
Cross-Species Differences in Analogical Ability

Object matches are easy; relational matches are hard

Matching-to-sample Task

- Rats, monkeys, apes, pigeons etc. readily learn to match objects
- But even chimps fail to learn relational match after hundreds of trials (Premack, 1983; Thompson & Oden)
1. Chimps readily succeed at object matches but not relational matches

2. Then, chimps are taught tokens for *same* and *different*

3. Chimps with this label training can succeed at relational matches
Hypothesis: Overt “same/different” labels induce internal “same/different” representations

Hard match

Recoded as

Relational match
Summary: Good News, Bad News, Good News

• Structure mapping processes can derive new learning by comparing cases

• But people often fail to access relevant analogs

• Relational retrieval can be facilitated by relational encoding
  • which can be facilitated by comparing examples

• Common relational language invites comparison
  • So humans can go beyond perceptual similarity
Conclusions

Humans are preeminent learners.

We are also preeminent analogizers

Part of our phenomenal learning ability comes from our structure-mapping abilities

“...the faculty for perceiving analogies is the best indication of genius.”

William James
Thank you

Collaborators:

Ken Forbus   Art Markman
Jeff Loewenstein Laura Namy
Kenneth Kurtz Phillip Wolff
Brian Bowdle Mary Jo Rattermann
Doug Medin Rob Goldstone
Sam Day Jason Jameson
Lera Boroditsky Ron Ferguson
Sven Kuehne Brian Falkenhainer
Leigh Thompson David Uttal
Stella Christie Flo Anggoro
Julie Colhoun
Collaborators

Ken Forbus  Ken Kurtz
Jeff Loewenstein  Laura Namy
Phillip Wolff  Art Markman
Stella Christie  Sven Kuehne
Leigh Thompson  Sam Day
Findings from Analogy Research

Surface remindings.
Analogical remindings are rare (especially among novices; remindings are mostly based on overall similarity or surface similarity).

Conservative learning.
Early learning often does not initially generalize much beyond starting exemplars.

Relational mapping.
Analogical comparison highlights common connected structure.

Similarity is like analogy.
Mundane literal similarity comparison behaves much like analogy: It conveys common relational structure AS WELL AS common features.

Progressive alignment
Easy concrete comparisons followed by more abstract comparisons -- a natural and effective learning sequence.
Why we’re so smart

• Humans dominate the planet not because of innate theories but because of exceptional learning ability

• Humans possess two great advantages over other intelligent species
  • Relational ability
  • Language

• This combination is not a coincidence
Mutual Facilitation of Relational Language and Relational Cognition

- Relational language supports relational cognition
  - Apes – the Relational Match to Sample Task
  - Children
    - Adding relational language helps
    - Lack of relational language hurts

- Relational comparison supports learning relational language
Some current projects

• Implicit analogy in adults: Inferences people don’t realize they’re making
  (Perrott, Bodenhausen & Gentner, 2005; Day & Gentner, 2007)

• Comparison in category formation and induction from categories (with Jason Jameson)

• Analogy in causal reasoning (with Julie Coulson)

• Origins of relational similarity in children (with Stella Christie)

• Comparative psychology of relational similarity in humans and apes (with Stella Christie, Nina Simms and Josep Call)

• Progressive alignment in learning relational categories (with Flo Anggoro)
Analogy leads to learning

- **Generalization:** Structural alignment highlights common relational system, thereby promoting
  - relational focus
    Gentner & Namy, 1999; Loewenstein & Gentner, 2001
  - relational abstraction and transfer
    Gentner, Loewenstein & Thompson, 2003; Gick & Holyoak, 1983; Kurtz, Miao & Gentner, 2001;

- **Selective inferences:** Analogy projects inferences connected to the common structure
  Clement & Gentner, 1991; Spellman & Holyoak, 1992; Markman, 1997

- **Selective differences:** Analogy highlights alignable differences
  -- differences connected to the common system
    e.g., different values on same predicate
    Gentner & Markman, 1994; Markman & Gentner, 1993
Three key points re comparison and generalization

1. Structural alignment process highlights common connected structure

2. Mundane literal similarity behaves much like analogy: Common relational structure becomes more salient (as do common object attributes)

   *Cool-aid is like water*
   *Heat is like water*

3. Progressive alignment learning sequence

   *Cool-aid/Water ➔ Heat/Water*
Analogical comparison $\rightarrow$ relational abstraction

- Align representations
- $1:1$ mappings
- Systematicity

- Highlight common structure

- Recognize schema in new situations
Summary

• Conservative early learning: Initial learning is typically context-bound, highly specific

• Similarity-based memory access to prior instances is strongly surface-driven (Gentner, Rattermann & Forbus, 1993; Holyoak & Koh, 1987; Ross, 1989)

• Comparing two analogous instances highlights common relational structure (Loewenstein, Thompson & Gentner, 1999; Gick & Holyoak, 1983)

• Comparing two literally similar instances ALSO highlights common structure (Gentner & Namy, 2000; Gentner, Anggoro & Klibanoff, in preparation)

• Progressive alignment: moving from close to far similarity is an effective learning sequence for novice learners
Further notes
Relational Language Fosters Relational Learning

Hearing a relational term applied to a pattern of relations invites:

**Stability**
- storing the pattern with its label
- preserving the schema
- makes schema more portable to new situations

**Reification**
- permits new assertions to be stated about it
- a named relational schema can be an argument to a higher-order proposition

**Symbolic Juxtaposition**
- comparing it with other situations with same label

**Uniform Relational Encoding**
- Habitual use of a set of relational terms promotes uniform relational encoding, which promotes reminding and transfer

Gentner & Loewenstein, 2001; Gentner & Rattermann, 1991; Loewenstein and Gentner, 1998 Kotovsky & Gentner, 1996
ORDER

Objects and constants are order 0. The order of a predicate is one plus the maximum of the order of its arguments.

- If x and y are objects, then GREATER-TTHAN (x, y) is first-order.
- AND CAUSE [GREATER-TTHAN (x, y), BREAK(x)] is second-order.

REPRESENTATIONAL TYPES

Entities: logical individuals: i.e., the objects and constants of a domain.

- Individual objects or beings; pieces of stuff; logical constants.

Operators = Predicates and Functions

Predicates (truth-functional) = Relations and Attributes

- Attributes: predicates that take one argument
  - Typical use: modifiers; properties of entities
  - Such as RED(apple) or SQUARE(table).

- Relations: predicates that take two or more arguments
  - First-order relations take entities as arguments
    - Typical use: events, comparisons, or states
    - Examples: HIT(ball, table) and INSIDE(ball, pocket).

  - Higher-order relations take other predicates as their arguments:
    - For example, CAUSE [HIT(cue stick, ball), ENTER(ball, pocket)].

Functions map one or more entities into another entity or constant.

- Typical use: dimensions or states
- Examples: SPEED(ball) maps ball into the quantity that describes its speed.
Is bootstrapping hypothesis selection? That is, are two distinct hypotheses being considered?

In progressive alignment most parsimonious hypothesis is that learner has an initially learned specific representation that is then abstracted to a more abstract [purely relational] representation.

In cases where one representation gives rise to another by dropping object properties, you don’t need to invoke ‘two hypotheses’ assumption.
KHOR studies

Demonstration of progressive alignment
Comparison → Re-representation & Discovery in 4-year-olds

Kotovsky & Gentner, 1996

Similarity Triads – No feedback

Study 1: 8 within-dim, 8 across-dim, random order

Note that relational choice is the only possible correct choice

Within-Dimension

- 68%

- Standard

- Relational Choice

- Non-relational Choice

Across-Dimension

- 48%

- (chance)

- Standard

- Relational Choice

- Non-relational Choice
Progressive alignment reveals higher-order abstract relations

**Study 2:** Same method: Similarity, no feedback, 4-year-olds

One change: Within-dim (concrete) triads first; then cross-dim triads

- 8 within-dim triads, Easily aligned
- 8 cross-dim triads

**Result:** 63% correct on cross-dim
Progressive alignment reveals higher-order abstract relations

Study 2: Same method: Similarity, no feedback, 4-year-olds
One change: Within-dim (concrete) triads first; then cross-dim triads

8 within-dim triads, Easily aligned
8 cross-dim triads

Result: 63% correct on cross-dim

“Even though the small one comes first and the big one’s in the middle, it’s exactly the same --- but different!”
8-yr-old: most prior cross-dim wrong, all subsequent right
Simulation of human processing of Analogy and similarity

SME – Structure-mapping Engine
Structure-mapping & Learning

Alignment → Projection
• Align analogs
• Derive common system
• Project inferences

Structure-mapping process suggests ways to learn

• Results don’t have to be known in advance
  • Starts with blind local matches
  • Global interpretations emerge from connections between local matches
    - So, unanticipated alignments
    - spontaneous candidate inferences
    - emergent abstractions
• Same process for overall literal similarity as for analogy
  • so infants don’t have to know much about relations to get started

This process offers a way to learn things you didn’t know you didn’t know
So far, result could be explained at purely featural level --

e.g., varied features of study set cancel out the single shape feature

Next study pits featural matches against relational match

If featural matches all pull in one direction, can comparison prompt responding in the other [relational] direction?
Structure-mapping in Analogy

Analogy entails structural alignment and inference

Analogy fosters a focus on connected relational structure

Analogy promotes structural abstraction
Analogy gives rise to structured abstractions

Invites abstraction and re-representation

- Wallcorp *divested* itself of Best Tires
- Martha *divorced* George
- → commonality: “They each *got rid of* something they no longer wanted”
Potential Challenge

Gentner & Namy concluded that children learned a new abstraction by comparing a bicycle and a tricycle.

But an alternative view is hypothesis selection:

Is it about circles? Or wheeled vehicles?

Must be wheeled vehicles
An analogy conveys that partly identical relational structures hold between objects in different domains. Corresponding objects need not resemble each other (easier if they do).

LS: Doe and fawn are like Mare and colt
AN: Hen and chick are like Mare and colt
An analogy conveys that partly identical relational structures hold between objects in different domains.

- Corresponding objects need not resemble each other (easier if they do)

  LS: Doe and fawn are like Mare and colt
  AN: Hen and chick are like Mare and colt

Implicit constraints:

- Structural consistency:
  - 1-1 correspondences
  - Parallel connectivity
- Systematicity: bias for connected structure during mapping
Analogical Comparison $\rightarrow$ Schema Abstraction $\rightarrow$ Transfer
(Summing across three studies)

N=219 pairs

Prop. Forming Contingent Contracts

Dyadic Schema Rating
Markman & Gentner (1993)

Similarity involves structural alignment

Proportion of Relational Responses

- 1MAP
- SIM -> 1MAP
Analogy supports abstraction of common relational structure

Structural abstraction

Theories of Similarity

Similarity as inverse of mental distance (Shephard)

Representations: Objects as points in mental space

Problems for Mental Distance Models (Tversky)

1. Asymmetries

   Hungary is like Russia  \(\leftrightarrow\)  Russia is like Hungary

2. Minimality  \(S(A,A)\)

   Distance = 0  \(\begin{array}{c|c} \circ & \circ \\ \hline \mbox{Distance = 0 for both} \end{array}\)

   Distance = 0  \(\begin{array}{c|c} \mbox{Yet, similarity seems} \\ \mbox{greater here} \end{array}\)
Alignable Differences in Conversation
Tiny Learning Opportunities

Eve Clark, CLS 1998
(from Jefferson, 1987)

Customer: Mm, the wales are wider apart than that.

Clerk: Okay, let me see if I can find one with wider threads.

Customer: Nope, the threads are wider apart than that.
Violation of Independence

Subjects rate how similar Standard (triangle figure) is to the other two figures.

- Adding same feature to all three figures should not change ranking of similarity to Standard.

A) Standard more similar to squares than to circles

\[ \triangle \text{ is like } \square \text{ or } \circ \]
Violation of Independence

Subjects rate how similar Standard (triangle figure) is to the other two figures.

- Adding same feature to all three figures should not change ranking of similarity to Standard.

A) Standard more similar to squares than to circles

\[ \triangle \text{ is like } \square \text{ or } \circ \]

B) Standard more similar to circles than to squares

\[ \triangle \text{ is like } \square \text{ or } \square \]
Alignable differences are highly salient

Nonalignable

Alignable

Gentner & Sagi, 2006
Results

- Same-different task:
  Dissimilar (N.A.) faster than similar (A.)

- Difference-identification task:
  Similar faster than dissimilar

Gentner & Sagi, 2006)
Progressive alignment in learning part-names

What helps children learn words for parts?

(e.g., leg, ear, wing)

Knowledge of object category (Markman; Mintz & Gleitman)

Point or act on part (Kobayashi)

Hypothesis: Structural alignment $\rightarrow$ Attention to commonalities and differences

$\rightarrow$ Ability to detect and name corresponding parts

- without knowledge of categories
- without E indicating parts
“This one has a blick.”

“Which one of these has a blick?”

“And which one of these has a blick?”
“This one has a blick.”

“Which one of these has a blick?”

“And which one of these has a blick?”
E1: Alignment in Learning Part Names

**Target:**
"This one has a blick."

**Test Pair:**
"Which one of these has a blick?"

![Diagram showing the comparison of low and high similarity (Lo Sim. and Hi Sim.) between different ages (3 yrs, 4 yrs, 5 yrs) and their proportion of correct responses.](image-url)
E2: Alignment in Differentiating Named Parts

**Target:**
“Which one of these has a blick?”

**Test Pair:**
“Which one of these has a blick?”

![Diagram with Low Sim. and High Sim. examples]

![Graph showing proportion sets correct for 3 yrs (n=42) and 4 yrs (n=37)]
E3: Progressive Alignment Condition

**Similarity**

- **High**
  - [Images of colored creatures]

- **Low**
  - [Images of colored creatures]
E3: Control Condition

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E3: First two trials vs. Last two trials

Proportion sets correct

Age

First two trials

Hi Sim

Lo Sim

Progressive Alignment

Last two trials

Lo Sim

Control

3-year-olds 4-year-olds 3-year-olds 4-year-olds

Chance