Appendix A

Designing High School Mathematics Courses Based on the Common Core Standards
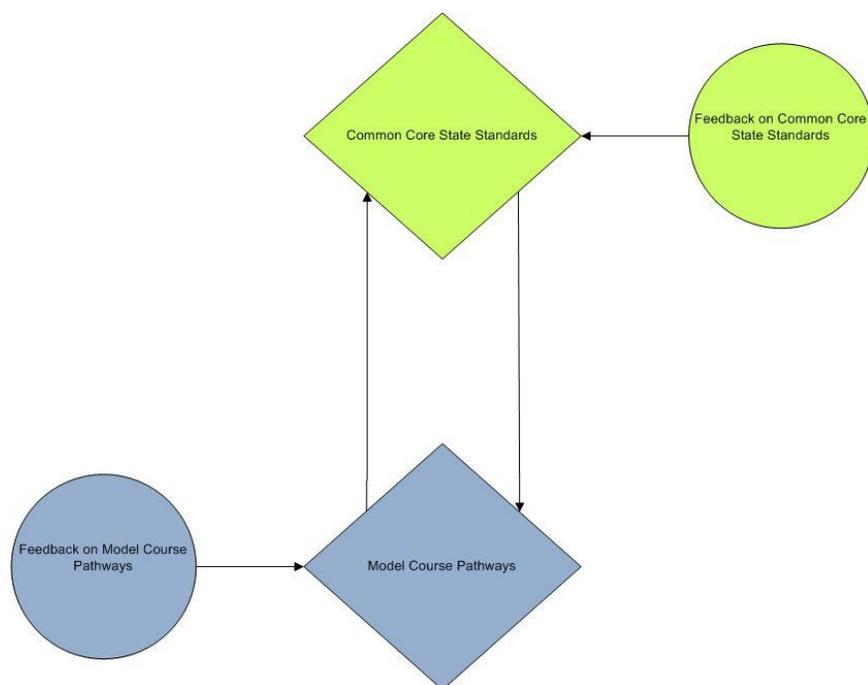
Overview:

The Common Core State Standards (CCSS) for Mathematics are organized by grade level in Grades K–8. At the high school level, the standards are organized by strand, showing a logical progression of content within those strands. As states consider how to implement the high school standards, a number have requested a model (or several models) that indicate how the CCSS content might be parsed into courses that provide a strong foundation for, and pathways to, college and work readiness and college credit bearing courses during the high school years. To address this request, Achieve (in partnership with the Common Core writing team) has convened a small team of experts, including state mathematics experts, teachers, mathematics faculty from two and four year institutions, and representatives from the workforce to develop Model Mathematics Course Pathways based on the Common Core State Standards.

The draft pathways and courses shown here are in an early stage of development. The public review period therefore provides a valuable opportunity for a wide range of stakeholders and experts to provide input on how to improve them.

This is an iterative process. As the Common Core State Standards undergo revisions, so do the courses. As the courses have been drafted, new insights about the standards have emerged, which has led to further refinements to the standards, which in turn has suggested new ways of thinking about the courses. In other words, the two processes have been iterative, and mutually reinforcing, and will continue to inform one another, as shown in the following flowchart.

For example, after a draft of the CCSS was produced in February, the Pathways underwent revision. These revisions led to additional changes to the CCSS, which will be included in the next draft of the Pathways. After the March public review period, feedback will inform additional edits to both the CCSS and the Pathways, where revisions to each will inform the other.

Three things are important to note:

1. The pathways and courses are models, not mandates. They illustrate possible approaches to organizing the content of the CCSS into coherent and rigorous courses that lead to college and work readiness as well as college credit bearing courses during the high school years.
2. The working documents shown here should not be regarded as final. They are included in this draft in order to promote a transparent process and provide opportunity for broad input and engagement. The final versions of these courses will change, as we receive feedback on them and on the standards upon which they are based.
3. The course descriptions are standards for what mathematics is covered in a course; they are not prescriptions for curriculum or pedagogy.

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Desired Feedback:
Readers are invited to submit feedback on the following questions:
- For each course:
  - Is it coherent? Is it logically ordered and is the focus clear?
  - Is each course manageable? In other words, would it seem to take one year or less to complete?
- For each sequence of courses:
  - Are the courses vertically articulated well? Is each sequence as a whole coherent?
- General Questions:
  - The reader will note that the courses and pathways have not been given titles. Should the typical terms such as "integrated" or "traditional" or "Algebra II" continue to be used, or is now the time to create new titles?
  - With respect to Pathway A, probability and statistics have been placed in each course. Should these standards exist in their own semester-long course, or continue to exist in each course throughout the pathway?
  - Do the expectations in the courses effectively balance the demands of:
    - Preparation for the workforce (Level 3 jobs);
    - Preparation for college (non-quantitative majors, STEM, and in-between);
    - International benchmarking;
    - The need to help students keep their options open for as long as possible?
  - What other changes could be made to make these sets of course standards stronger?

The Pathways:
At this time, two model course pathways are under construction:
- One an approach typically seen in the U.S. (Pathway A) that consists of two algebra courses and a geometry course, with some data, probability and statistics infused throughout each course (in the current design);
- The other an approach typically seen internationally (Pathway B) that consists of a sequence of 3 courses each of which treats aspects of algebra, geometry and data, probability, and statistics.

Both pathways assume four years of high school mathematics. Both pathways contain one course in each of years one and two, and then each pathway provides two options – or branches –for the third course, each of which leads to a variety of fourth year courses (see Chart 1). Many students, even most, should and will reach college and career readiness with time remaining for additional mathematics courses.
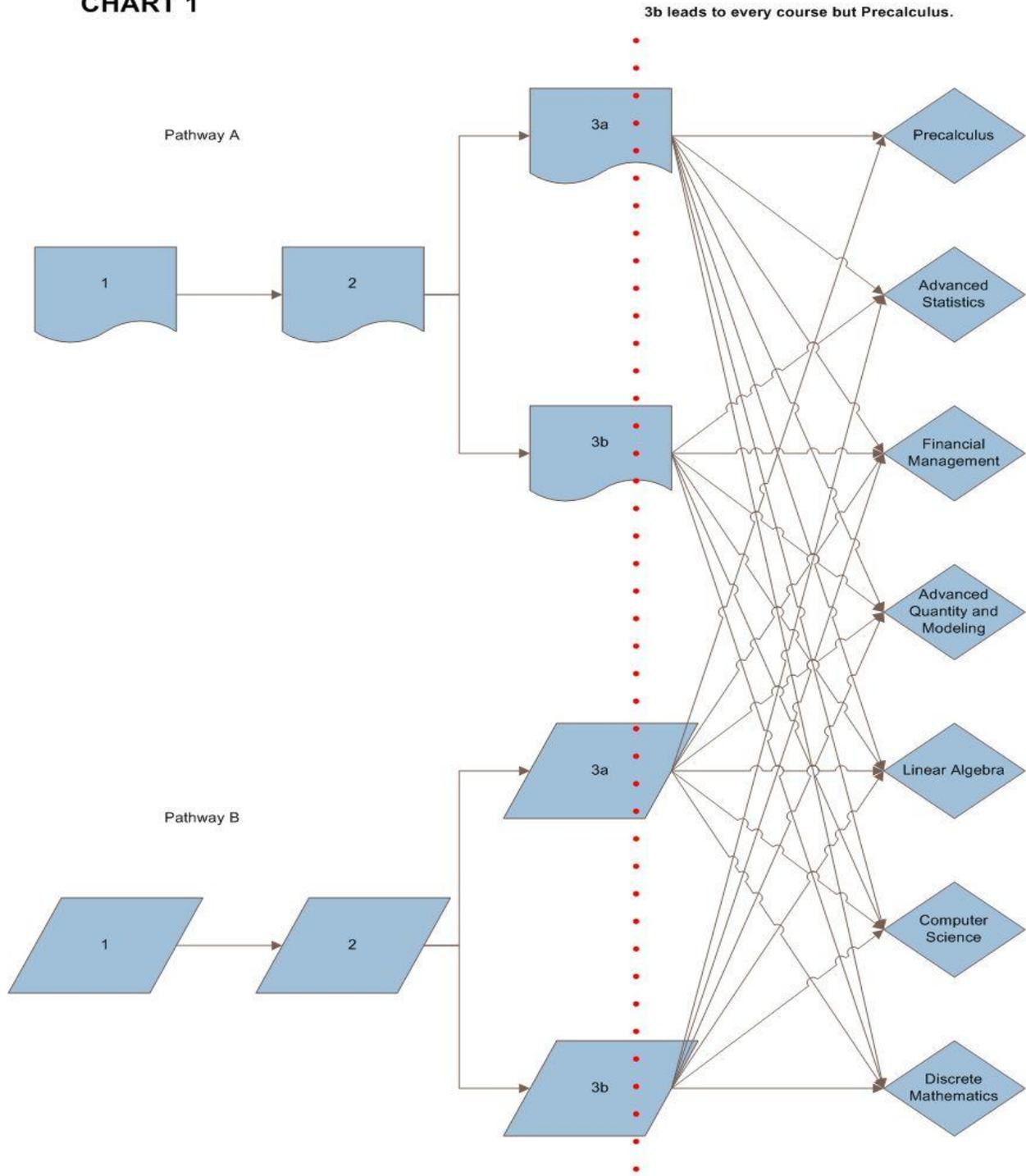
Both pathways lead directly to college opportunity, although students who know at an early age that they are interested in STEM majors and careers should endeavor to take even higher levels of mathematics in high school (e.g. Calculus by the senior year), and meet the college and career readiness level by the end of tenth grade. Those who reach the readiness level by the end of eleventh grade should take an additional mathematics course as seniors, with a subset of possibilities listed on the next page.

Each pathway should get the same attention to quality and resources including class sizes, teacher assignments and professional development, and materials. Indeed, these and other pathways should be avenues for students to pursue interests and aspirations. The following flow chart shows how the draft courses in the two pathways are sequenced, where the dotted line indicates the college and career readiness level.

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CHART 1

3b leads to every course but Precalculus.

Pathway A

1 → 2 → 3a
2 → 3b

Pathway B

1 → 2 → 3a
2 → 3b

Precalculus

Advanced Statistics

Financial Management

Advanced Quantity and Modeling

Linear Algebra

Computer Science

Discrete Mathematics

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Pathway A
Course 1

Building on their work with proportionality and linear equations in middle school, students turn their attention to the idea of function in this course. Students work closely with the expressions that define the functions, their graphs, domain, and function notation. Work with the functions is grounded in logical reasoning, where manipulations to the expressions are accomplished with intent and based on properties of arithmetic and the laws of equality. Students continue to expand and hone their abilities to model situations, including through the use of statistics, and to solve equations.

There are many unifying threads which wind their way throughout the course, offering greater focus and coherence. These threads are captured in the Unifying Standards, and should be applied appropriately to each instructional unit: Linear expressions, equations, and functions; Quadratic expressions, equations, and functions; and Exponential expressions, equations and functions. The final algebra unit in the course provides an opportunity to synthesize the year's work with functions by comparing and contrasting them.

Through this course, students continue to improve in their Mathematical Practices by attending to precision, constructing viable arguments and critiquing the reasoning of others, making sense of problems and persevering in solving them, looking for and making use of structure, looking for and expressing regularity in repeated reasoning, reasoning abstractly and quantitatively, modeling with mathematics, and using appropriate tools strategically. Teachers should look for opportunities to encourage student demonstration of their improvement of these practices.

Unifying Standards

Quantities* N.Q

N.Q.5	Use and interpret quantities and units correctly in algebraic formulas.
N.Q.6	Use and interpret quantities and units correctly in graphs and data displays (function graphs, data tables, scatter plots, and other visual displays of quantitative information). Generate graphs and data displays using technology. *This standard should be considered whenever a graph or data display is discussed.*[1]

Seeing Structure in Expressions A-SSE

A.SSE.1	Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression $1.15t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
A.SSE.2	Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.
A.SSE.3	Interpret an expression that represents a quantity in terms of the context. Include interpreting parts of an expression, such as terms, factors and coefficients. *
A.SSE.5	See expressions in different ways that suggest ways of transforming them. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

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* Standard with close connection to modeling

[1] Italicized text following a standard is an "instructional note" to the teacher.

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Creating Equations That Describe Numbers or Relationships — A-CED*

A.CED.1	Understand that equations in one variable are often created to describe properties of a specific but unknown number.
A.CED.2	Understand that equations in two or more variables that represent a relationship between quantities can be built by experimenting with specific numbers in the relationship.
A.CED.3	Write equations and inequalities that specify an unknown quantity or to express a relationship between two or more quantities. Use the equations and inequalities to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. *Note that "simple rational functions" in this course refers to functions of the form $f(x)=k/x$ ($x \neq 0$), unless otherwise noted. This function bears close connections to the linear function and is thus included in this course.*
A.CED.4	Rearrange formulas to highlight a quantity of interest. For example, transform Ohm's law V = IR to highlight resistance R; in motion with constant acceleration, transform $v_{f,x}^2 - v_{i,x}^2 = 2a_x(x_f - x_i)$ to highlight the change in position along the x-axis, $x_f - x_i$. *Focus on formula involving linear and quadratic expressions in this course. Emphasize the use of logical arguments based on the properties of equality and rules of arithmetic.*

Reasoning with Equations and Inequalities — A-REI

A.REI.1	Understand that to solve an equation algebraically, one makes logical deductions from the equality asserted by the equation, often in steps that replace it with a simpler equation whose solutions include the solutions of the original one. *Emphasize the use of logical arguments based on the properties of equality and rules of arithmetic.*
A.REI.4	Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane, often forming a curve or a line.
A.REI.5	Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
A.REI.8	Understand that equations and inequalities can be viewed as constraints in a problem situation, (e.g., inequalities describing nutritional and cost constraints on combinations of different foods.) *
A.REI.13	Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions. *In this course, focus on linear, simple rational, quadratic, and exponential functions. Include the use of technology to approximate intersections.*

Interpreting Functions — A-IF

The following standards support the introduction of each major function in this course.

A.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.
A.IF.3	Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function.
A.IF.4	Use function notation and evaluate functions for inputs in their domains.

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* Standard with close connection to modeling

A.IF.5	Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic).[★]
A.IF.6	Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities.[★]
A.IF.7	Compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, draw conclusions about the graph of a quadratic function from its algebraic expression.
A.IF.8	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.[★]
A.IF.10	Use technology to exhibit the effects of parameter changes on the graphs of linear, power, quadratic, square root, cube root, and polynomial functions, and simple rational, exponential, logarithmic, sine, cosine, absolute value and step functions.[★] *In this course, include linear, quadratic, simple rational functions of the form* $f(x) = \frac{a}{(x-h)} + k$ *and* $f(x) = \frac{a}{(x-h)^2} + k$, *exponential and absolute value functions.*

Building Functions A-BF

A.BF.1	Understand that functions can be described by specifying an explicit expression, a recursive process or steps for calculation.
A.BF.5	Write a function that describes a relationship between two quantities, for example by varying parameters in and combining standard function types (such as linear, quadratic or exponential functions). Experiment with parameters and illustrate an explanation of the behavior of the function when parameters vary using technology.[★]
A.BF.6	Solve problems involving linear, quadratic, and exponential functions.[★]
A.BF.7	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Focus on the linear (including absolute value), quadratic, and exponential functions.*

Linear, Quadratic, and Exponential Models A-LQE

A.LQE.9	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.[★]

Unit 1: Linear Expressions, Equations, and Functions

Quantities[★] NQ

N.Q.1	Understand that the magnitude of a quantity is independent of the unit used to measure it. For example, the density of a liquid does not change when it is measured in another unit. Rather, its measure changes. The chosen unit "measures" the quantity by giving it a numerical value ("the density of lead is 11.3 times that of water").
N.Q.2	Use units as a way to understand problems and to guide the solution of multi-step problems, involving, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game.

[★] Standard with close connection to modeling

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N.Q.3   Define metrics for the purpose of descriptive modeling. For example, find a good measure of overall highway safety; propose and debate measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled.

Reasoning with Equations and Inequalities A-REI

A.REI.3   Understand that given a system of two linear equations in two variables, adding a multiple of one equation to another produces a system with the same solutions. This principle, combined with principles already encountered with equations in one variable, allows for the simplification of systems.

Note that on the coordinate plane, the graph of a linear combination of two linear equations whose graphs intersect produces a third equation whose graph intersects the two original ones at their point of intersection.

A.REI.6   Understand that the solutions to a linear inequality in two variables can be graphed as a half-plane (excluding the boundary in the case of a strict inequality).

Include a variety of examples.

A.REI.7   Understand that solutions to several linear inequalities in two variables correspond to points in the intersection of the regions in the plane defined by the solutions to the inequalities.

G.GPE.1   Understand that two lines with well-defined slopes are perpendicular if and only if the product of their slopes is equal to -1.

A proof of this could be demonstrated using rotations of lines.

A.REI.11   Solve linear equations in one variable, including equations with coefficients represented by letters.

A.REI.14   Solve linear inequalities in one variable and graph the solution set on a number line.

A.REI.15   Solve systems of linear equations algebraically and graphically, focusing on pairs of linear equations in two variables.

Include situations where the two lines are perpendicular, and connect to the following standard.

G.GPE.7   Use the slope criteria for parallel and perpendicular lines to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

A.REI.17   Graph the solution set of a system of linear inequalities in two variables.

A.REI.18   In modeling situations, represent constraints by systems of equations and/or inequalities, and interpret solutions of these systems as viable or non-viable options in the modeling context.★

Focus on linear models in this unit.

Building Functions A-BF

A.BF.2   Understand that sequences are functions whose domain is a subset of the nonnegative integers.

Focus on linear/arithmetic sequences in this unit.

A.BF.8   Generate an arithmetic or geometric sequence given a recursive rule for the sequence.★

Focus on arithmetic sequences in this unit.

Linear, Quadratic, and Exponential Models A-LQE

A.LQE.1   Understand that a linear function, defined by $f(x) = mx + b$ for some constants m and b, models a situation in which a quantity changes at a constant rate, m, relative to another.★

A.LQE.5   Understand that in an arithmetic sequence, differences between consecutive terms form a constant sequence, and second differences are zero. Conversely, if the second differences are zero, the sequence is arithmetic. Arithmetic sequences can be seen as linear functions.★

A.LQE.10   Construct a function to describe a linear relationship between two quantities. Determine the rate of change and constant term of a linear function from a graph, a description of a relationship, or from two (x, y) values (including reading these from a table).★

A.LQE.13   Interpret the rate of change and constant term of a linear function or sequence in terms of the situation it models, and in terms of its graph or a table of values.★

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Unit 2: Statistical Analysis of Data

Summarizing Categorical and Quantitative Data S-SI

S.SI.1   Understand that statistical methods take variability into account to support making informed decisions based on data collected to answer specific questions.

S.SI.2   Understand that visual displays and summary statistics condense the information in data sets into usable knowledge.

S.SI.3   Understand that patterns of association or relationships between variables may emerge through careful analysis of multi-variable data.

Focus on determining whether potential linear or exponential relationships are suggested by the data. Connections should be made to understanding variability and how one might find useful patterns in it.

S.SI.6   Represent bivariate quantitative data on a scatter plot and describe how the variables are related.

Focus on bivariate data that exhibit linear or exponential relationships.

S.SI.7   Fit a linear function for scatter plots that suggest a linear association. Informally assess the fit of the model function by plotting and analyzing residuals.

Explain that one common measure of goodness of fit is the sum of squared vertical distances from the points to the line. The least squares line minimizes this sum. When the residuals (the vertical deviations from the line) are plotted against x, a pattern in the plot suggests that a linear function may not be the best way to describe the relationship between x and y.

S.SI.8   Use a model function fitted to the data to solve problems in the context of the data, interpreting the slope (rate of change) and the intercept (constant term).

S.SI.9   Compute (using technology) and interpret the correlation coefficient for a linear relationship between variables.

S.SI.10   Distinguish between correlation and causation.

Unit 3: Quadratic Expressions, Equations, and Functions

The Real Number System N-RN

N.RN.3   Understand that sums and products of rational numbers are rational.

N.RN.4   Understand that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational

Include definition of rational numbers as infinite and periodic decimals and definition of irrational numbers as infinite but non-periodic decimals.

Seeing Structure in Expressions A-SSE

A.SSE.4   Factor, expand, and complete the square in quadratic expressions.

Reasoning with Equations and Inequalities A-REI

A.REI.2   Understand that the method of completing the square can transform any quadratic equation in x into an equivalent equation of the form $(x - p)^2 = q$. This leads to the quadratic formula.

A.REI.12   Solve quadratic equations in one variable. Include methods such as inspection (e.g. for $x^2 = 49$), square roots, completing the square, the quadratic formula and factoring. Recognize when the quadratic formula gives complex solutions and write them as a $\pm$ bi for real numbers a and b.

Exclude complex solutions in this course.

A.REI.16   Solve algebraically a simple system consisting of one linear equation and one quadratic equation in two variables; for example, find points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Exclude quadratics that graph as circles in this course.

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| A.REI.18 | In modeling situations, represent constraints by systems of equations and/or inequalities, and interpret solutions of these systems as viable or non-viable options in the modeling context.★ |

Include situations with one quadratic and one line.

Interpreting Functions A-IF

| A.IF.2 | Understand that [quadratic] functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior. |
| A.IF.9 | Describe the qualitative behavior of [quadratic] functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★ |

Note that quadratics have one absolute maximum or one absolute minimum. Exclude notions of periodicity in this course.

| A.IF.11 | Transform quadratic polynomials algebraically to reveal different features of the function they define, such as zeros, extreme values, and symmetry of the graph. |

Linear, Quadratic, and Exponential Models A-LQE

A.LQE.2	Understand that quadratic functions have maximum or minimum values and can be used to model problems with optimum solutions.★
A.LQE.6	Understand that in a sequence that increases quadratically (e.g., $a_n = 3n^2 + 2n + 1$), differences between consecutive terms form an arithmetic sequence, and second differences form a constant sequence. Conversely, if the second differences form a constant sequence with non-zero value, the sequence increases quadratically.★
A.LQE.11	Use quadratic functions to model problems, e.g. in situations with optimum solutions.★

Unit 4: Exponential Expressions, Equations, and Functions

The Real Number System N-RN

N.RN.1	Understand that the laws of exponents for positive integer exponents follow from an understanding of exponents as indicating repeated multiplication, and from the associative law for multiplication.
N.RN.2	Understand that the definition of the meaning of zero, positive rational, and negative exponents follows from extending the laws of exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, since $(5^{1/3})^3 = 5^{(1/3)\cdot 3} = 5^1 = 5$, $5^{1/3}$ is a cube root of 5.
N.RN.5	Rewrite expressions using the laws of exponents. For example $(5^{1/2})^3 = 5^{3/2}$ and $1/5 = 5^{-1}$.

Quantities★ NQ

| N.Q.4 | Add, subtract, multiply, and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. |

Make connection between scientific notation's use of base 10 to the use of base 10 in solving exponential equations.

Seeing Structure in Expressions A-SSE

| A.SSE.6 | Rewrite expressions using the laws of exponents. For example, $(x^{1/2})^3 = x^{3/2}$ and $1/x = x^{-1}$. |

★ Standard with close connection to modeling

★ Standard with close connection to modeling

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A.SSE.7 — Use the laws of exponents to interpret expressions for exponential functions, recognizing positive rational exponents as indicating roots of the base and negative exponents as indicating the reciprocal of a power. For example, identify the per unit percentage change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and conclude whether it represents exponential growth or decay. Recognize that any non-zero number raised to the 0 power is 1 (for example, $12(1.05)^0 = 12$). Avoid common errors such as confusing $6(1.05)^t$ with $(6 \cdot 1.05)^t$ and $5(0.03)^t$ with $5(1.03)^t$.

Reasoning with Equations and Inequalities — A-REI

A.REI.19 — In the context of exponential models, solve equations of the form $a\, b^{ct} = d$ where a, c, and d are specific numbers and the base b is 2, 10, or e.★

In this course, use graphical methods or focus on equations having exact, integral solutions. For example, solve $2^x = 32$.

Interpreting Functions — A-IF

A.IF.2 — Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.

In this unit on exponential functions, focus on average rates of change (over intervals), whether the function is always increasing or always decreasing, and end behavior.

Building Functions — A-BF

A.BF.9 — As a way to describe routine modeling situations, write arithmetic and geometric sequences both recursively and in closed form, and translate between the two forms.★

Linear, Quadratic, and Exponential Models — A-LQE

A.LQE.3 — Understand that an exponential function, defined by $f(x) = ab^x$ or by $f(x) = a(1 + r)^x$ for some constants a, $b > 0$ and $r > -1$, models a situation where a quantity grows or decays by a constant factor or a constant percentage change over each unit interval.★

A.LQE.4 — Understand that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.★

A.LQE.7 — Understand that in a geometric sequence, ratios of consecutive terms are all the same.★

A.LQE.8 — Understand that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically or (more generally) as a polynomial function.★

Verify graphically and restrict polynomials to quadratics in this unit.

A.LQE.12 — Construct an exponential function in the form $f(x) = a(1 + r)^x$ or $f(x) = ab^x$ to describe a relationship in which one quantity grows with respect to another at a constant percent growth rate or a with a constant growth factor.★

Make connection to compound interest, here.

A,LQE.14 — Calculate and interpret the growth factor for an exponential function (presented symbolically or as a table) given a fixed time interval. Estimate the growth factor from a graph.★

A.LQE.15 — Recognize a quantitative relationship as linear, exponential or neither from description of a situation.★

A.LQE.16 — Compare quantities increasing exponentially to quantities increasing linearly or as a polynomial function.★

Restrict polynomials to quadratics in this part of the course.

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Pathway A
Course 2

Building on their work with lines, angles, and triangles in middle school, students turn their attention to a more formal treatment of geometry. The concepts of congruence, similarity and symmetry are developed through the concept of geometric transformation. Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. Applying a scale transformation to a geometric figure yields a similar figure. The transformation preserves angle measure, and lengths are related by a constant of proportionality. If the constant of proportionality is 1, distances are also preserved (so the transformation is a rigid transformation) and the figures are congruent.

Students move quickly to establish the triangle congruence theorems, and then apply them to other polygons. With congruence in hand, students examine the idea of similarity with a particular emphasis on triangles. Students then see additional applications of similarity including the definitions of sine, cosine and tangent for acute angles, which are founded on right triangle similarity. With the Pythagorean theorem, sine, cosine, and tangent are fundamental in many real-world and theoretical situations. With a strong understanding of the properties, attributes and relationships of triangles and polygons, students examine the geometry of circles, quickly bringing in connections to coordinate geometry.

Modeling continues to be an important activity in this course. With the knowledge of a rich body of theorems about geometric objects, their attributes and relationships, students can apply them in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Another application of geometry is to probability, where area models are provide additional insight and lead to a deeper study of probability concepts.

Students continue to improve in their Mathematical Practices by attending to precision, constructing viable arguments and critiquing the reasoning of others, making sense of problems and persevering in solving them, looking for and making use of structure, looking for and expressing regularity in repeated reasoning, reasoning abstractly and quantitatively, modeling with mathematics, and using appropriate tools strategically. Teachers should look for opportunities to encourage student demonstration of their improvement of these practices.

Unit 1: Triangles and Polygons — *Congruence*

Congruence   G-CO

G.CO.1	Understand that two geometric figures are congruent if there is a sequence of rigid motions (rotations, reflections, translations) that carries one onto the other. This is the principle of superposition.
G.CO.2	Understand that criteria for triangle congruence are ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent.
G.CO.3	Understand that criteria for triangle congruence (ASA, SAS, and SSS) can be established using rigid motions.
G.CO.4	Understand that geometric diagrams can be used to test conjectures and identify logical errors in fallacious proofs.
G.CO.5	Know and use (in reasoning and problem solving) definitions of angles, polygons, parallel and perpendicular lines, rigid motions, parallelograms and rectangles.

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G.CO.6   Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are exactly those equidistant from the segment's endpoints.*

G.CO.7   Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°, base angles of isosceles triangles are congruent, the triangle inequality, the longest side of a triangle faces the angle with the greatest measure and vice-versa, the exterior-angle inequality, and the segment joining midpoints of two sides of a triangle parallel to the third side and half the length.*

Connect back to logical reasoning about expressions and equations covered in previous course.

G.CO.8   Use and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid and kite.

G.CO.9   Characterize parallelograms in terms of equality of opposite sides, in terms of equality of opposite angles, and in terms of bisection of diagonals; characterize rectangles as parallelograms with equal diagonals.

G.CO.10   Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G.CO.11   Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.

G.CO.12   Use two-dimensional representations to transform figures and to predict the effect of translations, rotations and reflections.

G.CO.13   Use two-dimensional representations to transform figures and to predict the effect of dilations.

Unit 2: Triangles and Polygons — *Similarity, Right Triangles and Trigonometry*

Similarity, Right Triangles, and Trigonometry   G-SRT

G.SRT.1   Understand that dilating a line produces a line parallel to the original. (In particular, lines passing through the center of the dilation remain unchanged.)

G.SRT.2   Understand that the dilation of a given segment is parallel to the given segment and longer or shorter in the ratio given by the scale factor. A dilation leaves a segment unchanged if and only if the scale factor is 1.

G.SRT.3   Understand that the assumed properties of dilations can be used to establish the AA, SAS, and SSS criteria for similarity of triangles.

G.SRT.4   Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of sine, cosine and tangent.

G.SRT.5   Understand that a line parallel to one side of a triangle divides the other two proportionally, and conversely.

G.SRT.6   Use triangle similarity criteria to solve problems and to prove relationships in geometric figures. Include a proof of the Pythagorean Theorem using triangle similarity.

G.SRT.7   Use and explain the relationship between the sine and cosine of complementary angles.

G.SRT.8   Use sine, cosine, tangent and the Pythagorean Theorem to solve right triangles[2] in applied problems.

G.SRT.9   STEM Give an informal explanation using successive approximation that a dilation of scale factor r changes the length of a curve by a factor of r and the area of a region by a factor of r^2.

Trigonometry of General Triangles   G-TGT

G.TGT.1   STEM Understand that the formula A $=$ ½ ab sin(C) for the area of a triangle can be derived by drawing an auxiliary line from a vertex perpendicular to the opposite side. Applying this formula in three different ways leads to the Law of Sines.

G.TGT.2   STEM Understand that the Law of Cosines generalizes the Pythagorean Theorem.

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[2] A right triangle has five parameters, its three lengths and two acute angles. Given a length and any other parameter, "solving a right triangle" means finding the remaining three parameters.

G.TGT.4	STEM Understand that the Laws of Sines and Cosines embody the triangle congruence criteria, in that three pieces of information are usually sufficient to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that "Side-Side-Angle" is not a congruence criterion.
	Do not emphasize ambiguous case in this course. Constrain to a discussion of its existence.
G.TGT.5	STEM Explain proofs of the Law of Sines and the Law of Cosines.
	For proof, emphasize explanation.
G.TGT.6	STEM Use the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
	Do not include ambiguous case.

Unit 3: Circles

Circles G-C

G.CO.11	Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.

Include the construction of a circle, a circle inscribed in a triangle, and circumscribed by a triangle.

G.C.1	Understand that dilations can be used to show that all circles are similar.
G.C.2	Understand that here is a unique circle through three non-collinear points, or tangent to three non-concurrent lines.

Connect to the construction of a circle.

G.C.3	Identify and define radius, diameter, chord, tangent, secant and circumference.
G.C.4	Identify and describe relationships among angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
G.C.5	Determine the arc lengths and the areas of sectors of circles, using proportions.
G.C.6	STEM Construct a tangent line from a point outside a given circle to the circle.
G.C.7	STEM Prove and use theorems about circles, and use these theorems to solve problems. *Include:*

- *Symmetries of a circle*
- *Similarity of a circle to any other*
- *Tangent line, perpendicularity to a radius*
- *Inscribed angles in a circle, relationship to central angles, and equality of inscribed angles*
- *Properties of chords, tangents and secants as an application of triangle similarity.*

Unit 4: Expressing Geometric Properties with Equations

Expressing Geometric Properties with Equations G-GPE

G.GPE.2	Understand that the equation of a circle can be found using its definition and the Pythagorean Theorem.
G.GPE.3	Understand that transforming the graph of an equation by reflecting in the axes, translating parallel to the axes, or applying a dilation to one of the axes correspond to substitutions in the equation. *For example, reflection in the y axis corresponds to $(x,y) \rightarrow (-x,y)$, translation vertically down by three units corresponds to $(x,y) \rightarrow (x,y+3)$, and dilating by a factor of 2 parallel to the x-axis corresponds to $(x, y) \rightarrow (x/2, y)$.*
G.GPE.8	Find the point on the segment between two given points that divides the segment in a given ratio.

G.GPE.9   Use coordinates to compute perimeters of polygons and areas for triangles and rectangles, e.g. using the distance formula.★

G.GPE.10   Decide whether a point with given coordinates lies on a circle defined by a given equation.

G.GPE.11   Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).

G.GPE.12   Complete the square to find the center and radius of a circle given by an equation.

Include the case where the radius is 1 with center at the origin, and an introductory discussion of the unit circle.

Unit 5: Geometric Measurement, Dimension, and Modeling

Geometric Measurement and Dimension   G-GMD

G.GMD.1   Understand that the area of a decomposed figure is the sum of the areas of its components and is independent of the choice of dissection.

G.GMD.3   STEM Understand that Cavalieri's principle allows one to understand volume formulas informally by visualizing volumes as stacks of thin slices.

G.GMD.4   Find areas of polygons by dissecting them into triangles.

G.GMD.5   Explain why the volume of a cylinder is the area of the base times the height, using informal arguments.

G.GMD.6   For a pyramid or a cone, give a heuristic argument to show why its volume is one-third of its height times the area of its base.

G.GMD.7   Apply formulas and solve problems involving volume and surface area of right prisms, right circular cylinders, right pyramids, cones, spheres and composite figures.

Modeling with Geometry   G-MG

G.MG.1   Understand that models of objects and structures can be built from a library of standard shapes; a single kind of shape can model seemingly different objects.★

G.MG.2   Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso or as a cylinder).★

G.MG.3   Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

G.MG.4   Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy constraints or minimize cost; working with typograph.

Unit 6: Probability Models

Significant emphasis should be given to the connection between probability and geometry. For example, there are geometry models for probability, including scaled two-way tables, for example. There are also true geometric probability problems, such as "What is the probability of breaking a piece of 10" linguine into two parts and having one part be more than 8" long?" Both types of models provide valuable opportunities to connect probability and geometry.

Probability Models   S-PM

★ Standard with close connection to modeling

★ Standard with close connection to modeling

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S.PM.1   Understand that in a probability model, individual outcomes have probabilities that sum to 1. When outcomes are categorized, the probability of a given type of outcome is the sum of the probabilities of all the individual outcomes of that type.

S.PM.2   Understand that uniform probability models are useful models for processes such as (i) the selection of a person from a population; (ii) the selection of a number in a lottery; (iii) any physical situation in which symmetry suggests that different individual outcomes are equally likely.

S.PM.3   Understand that two different empirical probability models for the same process will rarely assign exactly the same probability to a given type of outcome. But if the data sets are large and the methods used to collect the data for the two data sets are consistent, the agreement between the models is likely to be reasonably good.

S.PM.4   Understand that a (theoretical) uniform probability model may be judged by comparing it to an empirical probability model for the same process. If the theoretical assumptions are appropriate and the data set is large, then the two models should agree approximately. If the agreement is not good, then it may be necessary to modify the assumptions underlying the theoretical model or look for factors that might have affected the data used to create the empirical model.

S.PM.5   Use a uniform probability model to compute probabilities for a process involving uncertainty, including the random selection of a person from a population and physical situations where symmetry suggests that different individual outcomes are equally likely.

List the individual outcomes to create a sample space.

Label the individual outcomes in the sample space to reflect important characteristics or quantities associated with them.

Determine probabilities of individual outcomes, and determine the probability of a type or category of outcome as the fraction of individual outcomes it includes.

S.PM.6   Generate data by sampling, repeated experimental trials, and simulations. Record and appropriately label such data, and use them to construct an empirical probability model. Compute probabilities in such models.

S.PM.7   Compare probabilities from a theoretical model to probabilities from a corresponding empirical model for the same situation. If the agreement is not good, explain possible sources of the discrepancies.

Independently Combined Probability Models S-IPM

S.IPM.1   Understand that to describe a pair of random processes (such as tossing a coin and rolling a number cube), or one random process repeated twice (such as randomly selecting a student in the class on two different days), two probability models can be combined into a single model.

- The sample space for the combined model is formed by listing all possible ordered pairs that combine an individual outcome from the first model with an individual outcome from the second. Each ordered pair is an individual outcome in the combined model.

- The total number of individual outcomes (ordered pairs) in the combined model is the product of the number of individual outcomes in each of the two original models.

S.IPM.2   Understand that when two probability models are combined independently, the probability that one type of outcome in the first model occurs together with another type of outcome in the second model is the product of the two corresponding probabilities in the original models (the Multiplication Rule).

S.IPM.3   Combine two uniform models independently to compute probabilities for a pair of random processes (e.g., flipping a coin twice, selecting one person from each of two classes).

Use organized lists, tables and tree diagrams to represent the combined sample space.

Determine probabilities of ordered pairs in the combined model, and determine the probability of a particular type or category of outcomes in the combined model, as the fraction of ordered pairs corresponding to it.

S.IPM.4   For two independently combined uniform models, use the Multiplication Rule to determine probabilities.

Conditional Probability and the Laws of Probability S-CP

S.CP.1   Understand that events are subsets of a sample space; often, events of interest are defined by using characteristics (or categories) of the sample points, or as unions, intersections, or complements thereof ("and," "or," "not"). A sample point may belong to several events (categories).

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S.CP.2   Understand that if A and B are two events, then in a uniform model the conditional probability of A given B, denoted by $P(A \mid B)$, is the fraction of B's sample points that also lie in A.

S.CP.3   Understand that the laws of probability allow one to use known probabilities to determine other probabilities of interest.

S.CP.4   Compute probabilities by constructing and analyzing sample spaces, representing them by tree diagrams, systematic lists, and Venn diagrams.

S.C.5   Use the laws of probability to compute probabilities.

S.CP.6   Apply concepts such as intersections, unions and complements of events, and conditional probability and independence to define or analyze events, calculate probabilities and solve problems.

S.CP.7   Construct and interpret two-way tables to show probabilities when two characteristics (or categories) are associated with each sample point. Use a two-way table to determine conditional probabilities.★

S.CP.8   Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.★

S.CP.9   Use permutations and combinations to compute probabilities of compound events and solve problems.

★ Standard with close connection to modeling

Appendix A

Designing High School Mathematics Courses Based on the Common Core Standards

Pathway A
Course 3a

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of mathematical structures to include polynomial, rational[3] square root, and cube root functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

There are many unifying threads which wind their way throughout the course, offering greater focus and coherence. These threads are captured in the Unifying Standards, and should be applied appropriately to each instructional unit. The final unit in this course provides students opportunities to engage in inferential reasoning through further study in statistics.

Through this course, students continue to improve in their Mathematical Practices by attending to precision, constructing viable arguments and critiquing the reasoning of others, making sense of problems and persevering in solving them, looking for and making use of structure, looking for and expressing regularity in repeated reasoning, reasoning abstractly and quantitatively, modeling with mathematics, and using appropriate tools strategically. Teachers should look for opportunities to encourage student demonstration of their improvement of these practices.

Unifying Standards

Seeing Structure in Expressions   A-SSE

A.SSE.1	Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
A.SSE.2	Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.
N.Q.5	Use and interpret quantities and units correctly in algebraic formulas.
A.SSE.3	Interpret an expression that represents a quantity in terms of the context. Include interpreting parts of an expression, such as terms, factors and coefficients. ★
A.SSE.5	See expressions in different ways that suggest ways of transforming them. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Reasoning with Equations and Inequalities   A-REI

A.REI.1	Understand that to solve an equation algebraically, one makes logical deductions from the equality asserted by the equation, often in steps that replace it with a simpler equation whose solutions include the solutions of the original one. *Emphasize the use of logical arguments based on the properties of equality, rules of arithmetic, and, where appropriate, the laws of exponents.*
A.REI.4	Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane, often forming a curve or a line.

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[3] In this course "rational" expressions are taken to be those that are at most linear polynomials divided by quadratic polynomials.

A.REI.5	Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
A.REI.13	Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions.
	Include the use of technology to approximate intersections.

Interpreting Functions A-IF

The following standards support the introduction of each major function (polynomial, rational, square root, and cube root) in this course.

A.IF.2	Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.
A.IF.3	Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function.
A.IF.4	Use function notation and evaluate functions for inputs in their domains.
A.IF.5	Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic). ★
A.IF.6	Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities.★
A.IF.7	Compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, draw conclusions about the graph of a quadratic function from its algebraic expression.
A.IF.8	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
A.IF.9	Describe the qualitative behavior of functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
A.IF.10	Use technology to exhibit the effects of parameter changes on the graphs of linear, power, quadratic, square root, cube root, and polynomial functions, and simple rational, exponential, logarithmic, sine, cosine, absolute value and step functions.★
	While the focus of this standard is on the effects of parameter changes on the graphs of several functions, it is advisable to first introduce the graphs through brief examinations of the functions' domains.
	For logarithmic, absolute value and step functions, demonstrate the genesis of their graphs by evaluating the functions for several inputs in their domains and graphing the results.
	While the trigonometric functions will be fully treated in the Precalculus course, one may describe the genesis of their graphs for the purpose of studying transformations. This can be done by demonstrating the use of benchmark acute angles with vertex at the origin and right angles formed with the x-axis and points on the unit circle to develop a table of values in the first quadrant. Demonstrate the use transformations to move the angles to subsequent quadrants eventually generalizing to all quadrants to continue the table. Use the tables to generate the graph of sine and cosine.

Building Functions A-BF

★ Standard with close connection to modeling

A.BF.1	Understand that functions can be described by specifying an explicit expression, a recursive process or steps for calculation.
A.BF.5	Write a function that describes a relationship between two quantities, for example by varying parameters in and combining standard function types (such as linear, quadratic or exponential functions). Experiment with parameters and illustrate an explanation of the behavior of the function when parameters vary using technology. ★
	Include non-traditional functions, as well. Students should become adept at writing functions to represent a variety of situations.
A.BF.7	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
	Focus on linear, exponential, rational, quadratic, square root, and cube root functions; consider examples from other types of functions. Note the common effect of each of these transformations across function types.

Linear, Quadratic, and Exponential Models   A-LQE

A.LQE.9	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

Unit 1: Polynomial and Radical Expressions, Equations, and Functions

The Complex Number System   N-CN

N.CN.1	Understand that the relation $i^2 = -1$ and the commutative, associative, and distributive laws can be used to calculate with complex numbers.
	Include discussion of the extension of the number system to complex numbers based on need to solve certain equations.
N.CN.2	STEM Understand that polynomials can be factored over the complex numbers, e.g., as in $x^2 + 4 = (x + 2i)(x - 2i)$.
	Extend to higher degree polynomials where factoring is reasonable. Include application of factoring polynomials where factoring is reasonable. Include application of factoring polynomials to reduce rational expressions to lowest terms.
N.CN.6	Add, subtract, and multiply complex numbers.
	Extend beyond the application to quadratics and $i^2 = -1$ to include cubics and higher powers of i.
N.CN.8	STEM Solve quadratic equations with real coefficients that have complex solutions.

Seeing Structure in Expressions   A-SSE

A.SSE.6	Rewrite expressions using the laws of exponents. For example, $(x^{1/2})^3 = x^{3/2}$ and $1/x = x^{-1}$.

Reasoning with Equations and Inequalities   A-REI

A.REI.10	Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions.
	Focus on square root and cube root[4] functions in this unit.
A.REI.12	Solve quadratic equations in one variable. *Include methods such as inspection (e.g. for $x^2 = 49$), square roots, completing the square, the quadratic formula and factoring. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
	Focus on complex solutions in this course. Include connection of quadratic functions to partial sums of arithmetic series; emphasize derivation of the formula to find partial sums.

Arithmetic with Polynomials and Rational Expressions   A-APR

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[4] The introduction of the square root and cube root functions should be handled with care. In particular, their problematic nature over the complex numbers should be discussed as existing and to be encountered fully in later mathematics courses.

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A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.
A.APR.2	Understand that polynomial identities become true statements no matter which real numbers are substituted. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
	Extend beyond quadratic examples to include higher degree polynomials in this course.
A.APR.3	Understand the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
	Relate long division of polynomials to long division of positive integers.
A.APR.4	STEM Understand that the Binomial Theorem gives the expansion of $(x + a)^n$ in powers of x for a positive integer n and a real number a, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.
	Connect to probability and Pascal's triangle. The proof may be explained by the instructor, but is not intended to be performed by students on their own.
A.APR.6	Add, subtract and multiply polynomials.
	Emphasize relationship of polynomial multiplication as an application of the distributive property.
A.APR.7	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the polynomial.

Unit 2: Rational and Exponential Expressions, Equations, and Functions

Arithmetic with Polynomials and Rational Expressions   A-APR

A.APR.5	STEM Understand that rational expressions are quotients of polynomials. They form a system analogous to the rational numbers, closed under division by a nonzero rational function.
A.APR.8	Transform simple rational expressions using the commutative, associative, and distributive laws, and the inverse relationship between multiplication and division.
	Relate to division by zero. Clarify this it is the function output value that becomes zero with polynomial division. Distinguish between the output variable and the input value of the function that makes the function equal zero.
A.APR.9	Divide a polynomial $p(x)$ by a divisor of the form $x - a$ using long division.
A.APR.11	STEM Identify zeros and asymptotes of rational functions, when suitable factorizations are available, and use the zeros and asymptotes to construct a rough graph of the function.
A.APR.12	STEM Divide polynomials, using long division for linear divisors and long division or a computer algebra system for higher degree divisors.

Reasoning with Equations and Inequalities   A-REI

A.REI.10	Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions.
	Focus on rational functions in this unit.
A.REI.20	STEM Relate the properties of logarithms to the laws of exponents and solve equations involving exponential functions.

Unit 3: Statistics and Probability

Making Inferences and Justifying Conclusions   S-IC

S.IC.1	Understand that statistics is a process for making inferences about population parameters based on a sample from that population; randomness is the foundation for statistical inference.

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S.IC.2   Understand that the design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.

S.IC.3   Understand that simulation-based techniques are powerful tools for making inferences and justifying conclusions from data.

S.IC.4   Use probabilistic reasoning to decide if a specified model is consistent with results from a given data-generating process. (For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?)

S.IC.5   Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each.

S.IC.6   Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.7   Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment.

S.IC.8   Evaluate reports based on data.

Experimenting and Simulating to Model Probabilities   S-ES

S.ES.1   Understand that sets of data obtained from surveys, simulations or other means can be used as probability models, by treating the data set itself as a sample space, in which the sample points are the individual pieces of data.

S.ES.2   Understand that the probability of an outcome can be interpreted as an assertion about the long-run proportion of the outcome's occurrence if the random experiment is repeated a large number of times.

S.ES.3   Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.

S.ES.4   Compare the results of simulations with predicted probabilities. When there are substantial discrepancies between predicted and observed probabilities, explain them.

S.ES.5   Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.

Using Probability to Make Decisions   S-MD

S.MD.1   Understand that the expected value of a random variable is the weighted average of its possible values, with weights given by their respective probabilities.

S.MD.2   Understand that when the possible outcomes of a decision can be assigned probabilities and payoff values, the decision can be analyzed as a random variable with an expected value, e.g. of an investment.

S.MD.3   Calculate expected value, e.g. to determine the fair price of an investment.

S.MD.4   Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S.MD.5   Evaluate and compare two investments or strategies with the same expected value, where one investment or strategy is safer than the other.

S.MD.6   Evaluate and compare two investments or strategies, where one investment or strategy is safer but has lower expected value. Include large and small investments, and situations with serious consequences.

S.MD.7   Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).

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Pathway A
Course 3b

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of mathematical structures to include polynomial, rational[5] square root, and cube root functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations. Additional emphasis is placed in this course on modeling. Students should engage with meaningful and rigorous modeling tasks to deepen their understanding of the mathematics.

There are many unifying threads which wind their way throughout the course, offering greater focus and coherence. These threads are captured in the Unifying Standards, and should be applied appropriately to each instructional unit. The final unit in this course provides students opportunities to engage in inferential reasoning through further study in statistics.

Through this course, students continue to improve in their Mathematical Practices by attending to precision, constructing viable arguments and critiquing the reasoning of others, making sense of problems and persevering in solving them, looking for and making use of structure, looking for and expressing regularity in repeated reasoning, reasoning abstractly and quantitatively, modeling with mathematics, and using appropriate tools strategically. Teachers should look for opportunities to encourage student demonstration of their improvement of these practices.

Unifying Standards

Seeing Structure in Expressions · A-SSE

Core Standards · Students understand that:

A.SSE.1 Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. *Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

A.SSE.2 Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.

N.Q.5 Use and interpret quantities and units correctly in algebraic formulas.

A.SSE.3 Interpret an expression that represents a quantity in terms of the context. Include interpreting parts of an expression, such as terms, factors and coefficients. ★

A.SSE.5 See expressions in different ways that suggest ways of transforming them. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Reasoning with Equations and Inequalities · A-REI

A.REI.1 Understand that to solve an equation algebraically, one makes logical deductions from the equality asserted by the equation, often in steps that replace it with a simpler equation whose solutions include the solutions of the original one.
Emphasize the use of logical arguments based on the properties of equality, rules of arithmetic, and, where appropriate, the laws of exponents.

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[5] In this course "rational" expressions are taken to be those that are at most linear polynomials divided by quadratic polynomials.

Appendix A

Designing High School Mathematics Courses Based on the Common Core Standards

A.REI.4	Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane, often forming a curve or a line.
A.REI.5	Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
A.REI.13	Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions.

Interpreting Functions A-IF

The following standards support the introduction of each major function (polynomial, rational, square root, and cube root) in this course.

A.IF.2	Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.
A.IF.3	Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function.
A.IF.4	Use function notation and evaluate functions for inputs in their domains.
A.IF.5	Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic). ★
A.IF.6	Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities. ★
A.IF.7	Compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, draw conclusions about the graph of a quadratic function from its algebraic expression.*
A.IF.8	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
A.IF.9	Describe the qualitative behavior of functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
A.IF.10	Use technology to exhibit the effects of parameter changes on the graphs of linear, power, quadratic, square root, cube root, and polynomial functions, and simple rational, exponential, logarithmic, sine, cosine, absolute value and step functions. ★ *While the focus of this standard is on the effects of parameter changes on the graphs of several functions, it is advisable to first introduce the graphs through brief examinations of the functions' domains.* *For logarithmic, absolute value and step functions, demonstrate the genesis of their graphs by evaluating the functions for several inputs in their domains and graphing the results.* *While the trigonometric functions will be fully treated in the Precalculus course, one may describe the genesis of their graphs for the purpose of studying transformations. This can be done by demonstrating the use of benchmark acute angles with vertex at the origin and right angles formed with the x-axis and points on the unit circle to develop a table of values in the first quadrant. Demonstrate the use transformations to move the angles to subsequent quadrants eventually generalizing to all quadrants to continue the table. Use the tables to generate the graph of sine and cosine.*

Building Functions A-BF

★ Standard with close connection to modeling

Appendix A

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A.BF.1   Understand that functions can be described by specifying an explicit expression, a recursive process or steps for calculation.

A.BF.5   Write a function that describes a relationship between two quantities, for example by varying parameters in and combining standard function types (such as linear, quadratic or exponential functions). Experiment with parameters and illustrate an explanation of the behavior of the function when parameters vary using technology. [★]

Include non-traditional functions, as well. Students should become adept at writing functions to represent a variety of situations.

A.BF.7   Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Focus on linear, exponential, rational, quadratic, square root, and cube root functions; consider examples from other types of functions. Note the common effect of each of these transformations across function types.

Linear, Quadratic, and Exponential Models   A-LQE

A.LQE.9   Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. [★]

__Modeling__

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car
- Modeling savings account balance, bacterial colony growth, or investment growth
- Critical path analysis, e.g. applied to turnaround of an aircraft at an airport.
- Risk situations, like extreme sports, pandemics and terrorism.
- Relating population statistics to individual predictions

In situation like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among
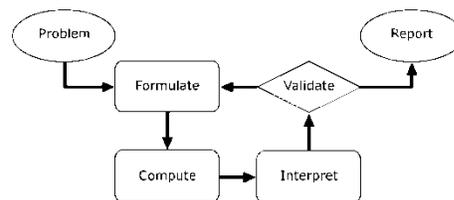
them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those than represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then, either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, CAS environments, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Unit 1: Polynomial and Radical Expressions, Equations, and Functions

The Complex Number System N-CN

N.CN.1   Understand that the relation $i^2 = -1$ and the commutative, associative, and distributive laws can be used to calculate with complex numbers.

Include discussion of the extension of the number system to complex numbers based on need to solve certain equations.

N.CN.6   Add, subtract, and multiply complex numbers.

Include application to quadratics and extend to include cubics and higher powers of i.

Seeing Structure in Expressions A-SSE

A.SSE.6   Rewrite expressions using the laws of exponents. For example, $(x^{1/2})^3 = x^{3/2}$ and $1/x = x^{-1}$.

Reasoning with Equations and Inequalities A-REI

A.REI.10   Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions.

Focus on square root and cube root[6] functions in this unit.

A.REI.12   Solve quadratic equations in one variable. *Include methods such as inspection (e.g. for $x^2 = 49$), square roots, completing the square, the quadratic formula and factoring. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*

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[6] The introduction of the square root and cube root functions should be handled with care. In particular, their problematic nature over the complex numbers should be discussed as existing and to be encountered fully in later mathematics courses.

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Focus on complex solutions in this course. Include connection of quadratic functions to partial sums of arithmetic series; emphasize derivation of the formula to find partial sums.

Arithmetic with Polynomials and Rational Expressions A-APR

A.APR.1   Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.

A.APR.2   Understand that polynomial identities become true statements no matter which real numbers are substituted. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

       Extend beyond quadratic examples to include higher degree polynomials in this course.

A.APR.3   Understand the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

       Relate long division of polynomials to long division of positive integers.

A.APR.6   Add, subtract and multiply polynomials.

       Emphasize relationship of polynomial multiplication as an application of the distributive property.

A.APR.7   Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the polynomial.

A.APR.9   Divide a polynomial $p(x)$ by a divisor of the form $x - a$ using long division.

Unit 2: Rational and Exponential Expressions, Equations, and Functions

Arithmetic with Polynomials and Rational Expressions A-APR

A.APR.5   STEM Understand that rational expressions are quotients of polynomials. They form a system analogous to the rational numbers, closed under division by a nonzero rational function.

A.APR.8   Transform simple rational expressions using the commutative, associative, and distributive laws, and the inverse relationship between multiplication and division.

       Relate to division by zero. Clarify this it is the function output value that becomes zero with polynomial division. Distinguish between the output variable and the input value of the function that makes the function equal zero.

A.APR.9   Divide a polynomial $p(x)$ by a divisor of the form $x - a$ using long division.

Reasoning with Equations and Inequalities A-REI

A.REI.10   Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions.

       Focus on rational functions in this unit.

Unit 3: Statistics and Probability

Making Inferences and Justifying Conclusions S-IC

S.IC.1   Understand that statistics is a process for making inferences about population parameters based on a sample from that population; randomness is the foundation for statistical inference.

S.IC.2   Understand that the design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.

S.IC.3   Understand that simulation-based techniques are powerful tools for making inferences and justifying conclusions from data.

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S.IC.4	Use probabilistic reasoning to decide if a specified model is consistent with results from a given data-generating process. (For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?)
S.IC.5	Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each.
S.IC.6	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S.IC.7	Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment.
S.IC.8	Evaluate reports based on data.

Experimenting and Simulating to Model Probabilities S-ES

S.ES.1	Understand that sets of data obtained from surveys, simulations or other means can be used as probability models, by treating the data set itself as a sample space, in which the sample points are the individual pieces of data.
S.ES.2	Understand that the probability of an outcome can be interpreted as an assertion about the long-run proportion of the outcome's occurrence if the random experiment is repeated a large number of times.
S.ES.3	Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.
S.ES.4	Compare the results of simulations with predicted probabilities. When there are substantial discrepancies between predicted and observed probabilities, explain them.
S.ES.5	Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.

Using Probability to Make Decisions S-MD

S.MD.1	Understand that the expected value of a random variable is the weighted average of its possible values, with weights given by their respective probabilities.
S.MD.2	Understand that when the possible outcomes of a decision can be assigned probabilities and payoff values, the decision can be analyzed as a random variable with an expected value, e.g. of an investment.
S.MD.3	Calculate expected value, e.g. to determine the fair price of an investment.
S.MD.4	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
S.MD.5	Evaluate and compare two investments or strategies with the same expected value, where one investment or strategy is safer than the other.
S.MD.6	Evaluate and compare two investments or strategies, where one investment or strategy is safer but has lower expected value. Include large and small investments, and situations with serious consequences.
S.MD.7	Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).

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Pathway B
Course 1

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. The course is conceived in five units: (1) Relationships between quantities, (2) Linear and exponential functions, (3) Descriptive statistics, (4) Congruence and rigid motions, and (5) Connecting algebra and geometry through coordinates.

Relationships Between Quantities. This unit builds on informal experiences with functions, equations, inequalities, and their graphs. Students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They use average rates of change (over an interval) to describe nonlinear functions and to distinguish linear from nonlinear functions.   They explore functions graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students understand sequences as functions. They work with functions given by graphs, tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions are given through contexts, they reason with the units in which those quantities are measured.

Linear and Exponential Relationships. Students reason about the laws of counting-number exponents, and they extend the laws of exponents to the integers to determine the meanings of zero and negative exponents. Students compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions. They develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities and their graphical representations. They explore systems of equations and inequalities and find and interpret their solutions.

Statistical Analysis of Data. Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models, and with a linear model they look at residuals to analyze the goodness of fit.

Congruence and Rigid Motions. In previous grades, students were asked to draw triangles based on given measurements. In this course, based on analyses of rigid motions and formal constructions, triangle congruence theorems are established and then used to prove theorems and to solve problems about triangles, quadrilaterals, and other polygons.

Connecting Algebra and Geometry Through Coordinates. Students use a coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Unit 1: Relationships between Quantities

Quantities★   NQ

N.Q.1	Understand that the magnitude of a quantity is independent of the unit used to measure it. For example, the density of a liquid does not change when it is measured in another unit. Rather, its measure changes. The chosen unit "measures" the quantity by giving it a numerical value ("the density of lead is 11.3 times that of water").

★ Standard with close connection to modeling

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N.Q.2   Use units as a way to understand problems and to guide the solution of multi-step problems, involving, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game.

N.Q.3   Define metrics for the purpose of descriptive modeling. For example, find a good measure of overall highway safety; propose and debate measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled.

N.Q.4   Add, subtract, multiply, and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Make connection between scientific notation's use of base 10 to the use of base 10 in solving exponential equations.[7]

N.Q.5   Use and interpret quantities and units correctly in algebraic formulas.

N.Q.6   Use and interpret quantities and units correctly in graphs and data displays (function graphs, data tables, scatter plots, and other visual displays of quantitative information). Generate graphs and data displays using technology. *This standard should be considered whenever a graph or data display is discussed.*

Seeing Structure in Expressions   A-SSE

A.SSE.3   Interpret an expression that represents a quantity in terms of the context. Include interpreting parts of an expression, such as terms, factors and coefficients. ★

Creating Equations That Describe Numbers or Relationships   A-CED★

A.CED.2   Understand that equations in two or more variables that represent a relationship between quantities can be built by experimenting with specific numbers in the relationship.

A.CED.3   Write equations and inequalities that specify an unknown quantity or to express a relationship between two or more quantities. Use the equations and inequalities to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Focus on equations arising from situations involving linear and exponential functions in this course.

A.CED.4   Rearrange formulas to highlight a quantity of interest. *For example, transform Ohm's law $V = IR$ to highlight resistance R; in motion with constant acceleration, transform $v_{f,x}^2 - v_{i,x}^2 = 2a_x(x_f - x_i)$ to highlight the change in position along the x-axis, $x_f - x_i$.*
Focus on formula involving linear expressions in this course. Emphasize the use of logical arguments based on the properties of equality and rules of arithmetic.

Reasoning with Equations and Inequalities   A-REI

A.REI.1   Understand that to solve an equation algebraically, one makes logical deductions from the equality asserted by the equation, often in steps that replace it with a simpler equation whose solutions include the solutions of the original one.
Emphasize the use of logical arguments based on the properties of equality and rules of arithmetic.

A.REI.5   Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Focus on systems of linear equations in this course.

A.REI.7   Understand that solutions to several linear inequalities in two variables correspond to points in the intersection of the regions in the plane defined by the solutions to the inequalities.

A.REI.8   Understand that equations and inequalities can be viewed as constraints in a problem situation, (e.g., inequalities describing nutritional and cost constraints on combinations of different foods.) ★

In this course, focus on constraints that are linear or exponential.

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[7] Italicized text following a standard is an "instructional note" to the teacher.

★ Standard with close connection to modeling

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A.REI.13	Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions.
	In this course, focus on functions f(x) and g(x) that are linear or exponential functions.
A.REI.18	In modeling situations, represent constraints by systems of equations and/or inequalities, and interpret solutions of these systems as viable or non-viable options in the modeling context.★

Interpreting Functions A-IF

A.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.
	Introduced as a general concept but applied in terms of linear and exponential functions in this course.
A.IF.2	Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.
	Introduced as a general concept but applied in terms of linear and exponential functions in this course. Relate average rates of change to linear approximations and note different characteristics of exponential growth and decay.
A.IF.3	Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function.
	Focus on rewriting linear and exponential functions and on vertical translations in this course.
A.IF.4	Use function notation and evaluate functions for inputs in their domains.
	Focus on linear and exponential relationships in this course.
A.IF.5	Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic).★
	Focus on linear and exponential relationships in this course.
A.IF.6	Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities.★
	Focus on linear and exponential functions in this course.
A.IF.7	Compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, draw conclusions about the graph of a quadratic function from its algebraic expression.
	Focus on values and properties of linear and exponential functions in this course.
A.IF.8	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★
	Focus on linear and exponential functions in this course.
A.IF.9	Describe the qualitative behavior of functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
	Focus on the behavior of linear and exponential functions in this course.

Building Functions A-BF

A.BF.1	Understand that functions can be described by specifying an explicit expression, a recursive process or steps for calculation.
A.BF.2	Understand that sequences are functions whose domain is a subset of the nonnegative integers.

★ Standard with close connection to modeling

A.BF.5	Write a function that describes a relationship between two quantities, for example by varying parameters in and combining standard function types (such as linear, quadratic or exponential functions). Experiment with parameters and illustrate an explanation of the behavior of the function when parameters vary using technology. [★]

Discuss building functions from basic operations of arithmetic using one operation — $y = ax$, $y = x + a$, or $y = a - x$ and focus on linear and exponential functions in this course.

Linear, Quadratic, and Exponential Models {A-LQE}

A,LQE.14	Calculate and interpret the growth factor for an exponential function (presented symbolically or as a table) given a fixed time interval. Estimate the growth factor from a graph. [★]

Focus on linear and exponential functions in this course.

Unit 2: Linear and Exponential Relationships

The Real Number System {N-RN}

N.RN.1	Understand that the laws of exponents for positive integer exponents follow from an understanding of exponents as indicating repeated multiplication, and from the associative law for multiplication.

In this course, focus on integral exponents.

N.RN.2	Understand that the definition of the meaning of zero, positive rational, and negative exponents follows from extending the laws of exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, since $(5^{1/3})^3 = 5^{(1/3)\cdot 3} = 5^1 = 5$, $5^{1/3}$ is a cube root of 5.*

N.RN.5	Rewrite expressions using the laws of exponents. *For example $(5^{1/2})^3 = 5^{3/2}$ and $1/5 = 5^{-1}$.*

Seeing Structure in Expressions {A-SSE}

A.SSE.1	Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression $1.15t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

In this course, focus on different forms of linear functions.

A.SSE.2	Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.

In this course, focus on linear expressions.

A.SSE.3	Interpret an expression that represents a quantity in terms of the context. Include interpreting parts of an expression, such as terms, factors and coefficients. [★]

A.SSE.6	Rewrite expressions using the laws of exponents. For example, $(x^{1/2})^3 = x^{3/2}$ and $1/x = x^{-1}$.

In this course, focus on integer exponents.

Creating Equations That Describe Numbers or Relationships {A-CED [★]}

A.CED.1	Understand that equations in one variable are often created to describe properties of a specific but unknown number.

Reasoning with Equations and Inequalities {A-REI}

[★] Standard with close connection to modeling

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A.REI.3 Understand that given a system of two linear equations in two variables, adding a multiple of one equation to another produces a system with the same solutions. This principle, combined with principles already encountered with equations in one variable, allows for the simplification of systems.

Note that on the coordinate plane, the graph of a linear combination of two linear equations whose graphs intersect produces a third equation whose graph intersects the two original ones at their point of intersection.

A.REI.4 Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane, often forming a curve or a line.

Focus on linear and exponential equations in this course.

A.REI.6 Understand that the solutions to a linear inequality in two variables can be graphed as a half-plane (excluding the boundary in the case of a strict inequality).

A.REI.11 Solve linear equations in one variable, including equations with coefficients represented by letters.

A.REI.14 Solve linear inequalities in one variable and graph the solution set on a number line.

A.REI.15 Solve systems of linear equations algebraically and graphically, focusing on pairs of linear equations in two variables.

A.REI.17 Graph the solution set of a system of linear inequalities in two variables.

A.REI.19 In the context of exponential models, solve equations of the form $a\,b^{ct} = d$ where a, c, and d are specific numbers and the base b is 2, 10, or e.★

In this course, use graphical methods or focus on equations having exact, integral solutions. For example, solve $2^x = 32$.

Interpreting Functions A-IF

A.IF.2 Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.

Introduced as a general concept but applied in terms of linear and exponential functions in this course. Relate average rates of change to linear approximations and note different characteristics of exponential growth and decay.

A.IF.3 Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function.

Distinguish between linear and non-linear and focus on vertical translations in this course.

Building Functions A-BF

A.BF.2 Understand that sequences are functions whose domain is a subset of the nonnegative integers.

Note that arithmetic sequences are linear functions and geometric sequences are exponential functions.

A.BF.5 Write a function that describes a relationship between two quantities, for example by varying parameters in and combining standard function types (such as linear, quadratic or exponential functions). Experiment with parameters and illustrate an explanation of the behavior of the function when parameters vary using technology.★

Discuss building functions from basic operations of arithmetic using one operation — $y = ax$, $y = x + a$, or $y = a − x$ and focus on linear and exponential functions in this course.

A.BF.6 Solve problems involving linear, quadratic, and exponential functions.★

Focus on linear and exponential functions in this course.

A.BF.8 Generate an arithmetic or geometric sequence given a recursive rule for the sequence.★

Focus on arithmetic sequences in this unit.

A.BF.9 As a way to describe routine modeling situations, write arithmetic and geometric sequences both recursively and in closed form, and translate between the two forms.★

Focus on arithmetic sequences in this unit.

Linear, Quadratic, and Exponential Models A-LQE

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A.LQE.1   Understand that a linear function, defined by $f(x) = mx + b$ for some constants m and b, models a situation in which a quantity changes at a constant rate, m, relative to another.[*]

A.LQE.3   Understand that an exponential function, defined by $f(x) = ab^x$ or by $f(x) = a(1 + r)^x$ for some constants a, $b > 0$ and $r > -1$, models a situation where a quantity grows or decays by a constant factor or a constant percentage change over each unit interval.[*]

A.LQE.4   Understand that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.[*]

A.LQE.5   Understand that in an arithmetic sequence, differences between consecutive terms form a constant sequence, and second differences are zero. Conversely, if the second differences are zero, the sequence is arithmetic. Arithmetic sequences can be seen as linear functions.[*]

A.LQE.7   Understand that in a geometric sequence, ratios of consecutive terms are all the same.[*]

A.LQE.8   Understand that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically or (more generally) as a polynomial function.[*]

Compare exponential with linear only in this course.

A,LQE.14   Calculate and interpret the growth factor for an exponential function (presented symbolically or as a table) given a fixed time interval. Estimate the growth factor from a graph.[*]

Focus on linear and exponential functions in this course.

A.LQE.10   Construct a function to describe a linear relationship between two quantities. Determine the rate of change and constant term of a linear function from a graph, a description of a relationship, or from two (x, y) values (including reading these from a table).[*]

A.LQE.12   Construct an exponential function in the form $f(x) = a(1 + r)^x$ or $f(x) = ab^x$ to describe a relationship in which one quantity grows with respect to another at a constant percent growth rate or a with a constant growth factor.[*]

Make connection to compound interest, here.

A.LQE.13   Interpret the rate of change and constant term of a linear function or sequence in terms of the situation it models, and in terms of its graph or a table of values.[*]

A,LQE.14   Calculate and interpret the growth factor for an exponential function (presented symbolically or as a table) given a fixed time interval. Estimate the growth factor from a graph.[*]

A.LQE.15   Recognize a quantitative relationship as linear, exponential or neither from description of a situation.[*]

A.LQE.16   Compare quantities increasing exponentially to quantities increasing linearly or as a polynomial function.[*]

Exclude polynomial functions in this course.

Unit 3: Statistical Analysis of Data

Summarizing Categorical and Quantitative Data   S-SI

S.SI.1   Understand that statistical methods take variability into account to support making informed decisions based on data collected to answer specific questions.

S.SI.2   Understand that visual displays and summary statistics condense the information in data sets into usable knowledge.

S.SI.3   Understand that patterns of association or relationships between variables may emerge through careful analysis of multi-variable data.

In this course, focus on assessing whether potential linear or exponential relationships are indicated in this data.

S.SI.4   Summarize comparative or bivariate categorical data in two-way frequency tables. Interpret joint, marginal and conditional relative frequencies in the context of the data, recognizing possible associations and trends in bivariate categorical data.

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S.SI.5   Compare data on two or more count or measurement variables by using plots on the real number line (dot plots, histograms, and box plots). Use statistics appropriate to the shape of the data distribution to summarize center (median, mean) and spread (interquartile range, standard deviation) of the data sets. Interpret changes in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.SI.6   Represent bivariate quantitative data on a scatter plot and describe how the variables are related.

Focus on bivariate data that exhibit linear or exponential relationships.

S.SI.7   Fit a linear function for scatter plots that suggest a linear association. Informally assess the fit of the model function by plotting and analyzing residuals.

Explain that one common measure of goodness of fit is the sum of squared vertical distances from the points to the line. The least squares line minimizes this sum. When the residuals (the vertical deviations from the line) are plotted against x, a pattern in the plot suggests that a linear function may not be the best way to describe the relationship between x and y.

S.SI.8   Use a model function fitted to the data to solve problems in the context of the data, interpreting the slope (rate of change) and the intercept (constant term).

Focus on linear models in this course.

S.SI.9   Compute (using technology) and interpret the correlation coefficient for a linear relationship between variables.

S.SI.10   Distinguish between correlation and causation.

Probability Models   S-PM

S.PM.1   Understand that in a probability model, individual outcomes have probabilities that sum to 1. When outcomes are categorized, the probability of a given type of outcome is the sum of the probabilities of all the individual outcomes of that type.

S.PM.2   Understand that uniform probability models are useful models for processes such as (i) the selection of a person from a population; (ii) the selection of a number in a lottery; (iii) any physical situation in which symmetry suggests that different individual outcomes are equally likely.

S.PM.3   Understand that two different empirical probability models for the same process will rarely assign exactly the same probability to a given type of outcome. But if the data sets are large and the methods used to collect the data for the two data sets are consistent, the agreement between the models is likely to be reasonably good.

S.PM.4   Understand that a (theoretical) uniform probability model may be judged by comparing it to an empirical probability model for the same process. If the theoretical assumptions are appropriate and the data set is large, then the two models should agree approximately. If the agreement is not good, then it may be necessary to modify the assumptions underlying the theoretical model or look for factors that might have affected the data used to create the empirical model.

S.PM.5   Use a uniform probability model to compute probabilities for a process involving uncertainty, including the random selection of a person from a population and physical situations where symmetry suggests that different individual outcomes are equally likely.

- List the individual outcomes to create a sample space.

- Label the individual outcomes in the sample space to reflect important characteristics or quantities associated with them.

- Determine probabilities of individual outcomes, and determine the probability of a type or category of outcome as the fraction of individual outcomes it includes.

S.PM.6   Generate data by sampling, repeated experimental trials, and simulations. Record and appropriately label such data, and use them to construct an empirical probability model. Compute probabilities in such models.

S.PM.7   Compare probabilities from a theoretical model to probabilities from a corresponding empirical model for the same situation. If the agreement is not good, explain possible sources of the discrepancies.

Independently Combined Probability Models   S-IPM

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S.IPM.1   Understand that to describe a pair of random processes (such as tossing a coin and rolling a number cube), or one random process repeated twice (such as randomly selecting a student in the class on two different days), two probability models can be combined into a single model.

- The sample space for the combined model is formed by listing all possible ordered pairs that combine an individual outcome from the first model with an individual outcome from the second. Each ordered pair is an individual outcome in the combined model.

- The total number of individual outcomes (ordered pairs) in the combined model is the product of the number of individual outcomes in each of the two original models.

S.IPM.3   Combine two uniform models independently to compute probabilities for a pair of random processes (e.g., flipping a coin twice, selecting one person from each of two classes).

- Use organized lists, tables and tree diagrams to represent the combined sample space.

- Determine probabilities of ordered pairs in the combined model, and determine the probability of a particular type or category of outcomes in the combined model, as the fraction of ordered pairs corresponding to it.

Unit 4: Congruence and Rigid Motions

Congruence   G-CO

G.CO.1   Understand that two geometric figures are congruent if there is a sequence of rigid motions (rotations, reflections, translations) that carries one onto the other. This is the principle of superposition.

G.CO.2   Understand that criteria for triangle congruence are ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent.

G.CO.3   Understand that criteria for triangle congruence (ASA, SAS, and SSS) can be established using rigid motions.

G.CO.4   Understand that geometric diagrams can be used to test conjectures and identify logical errors in fallacious proofs.

G.CO.5   Know and use (in reasoning and problem solving) definitions of angles, polygons, parallel and perpendicular lines, rigid motions, parallelograms and rectangles.

G.CO.6   Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are exactly those equidistant from the segment's endpoints.*

G.CO.7   Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°, base angles of isosceles triangles are congruent, the triangle inequality, the longest side of a triangle faces the angle with the greatest measure and vice-versa, the exterior-angle inequality, and the segment joining midpoints of two sides of a triangle parallel to the third side and half the length.*

   Connect to logical reasoning about expressions and equations.

G.CO.8   Use and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid and kite.

G.CO.9   Characterize parallelograms in terms of equality of opposite sides, in terms of equality of opposite angles, and in terms of bisection of diagonals; characterize rectangles as parallelograms with equal diagonals.

G.CO.10   Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G.CO.11   Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.

G.CO.12   Use two-dimensional representations to transform figures and to predict the effect of translations, rotations and reflections.

Geometric Measurement and Dimension   G-GMD

G.GMD.1   Understand that the area of a decomposed figure is the sum of the areas of its components and is independent of the choice of dissection.

Unit 5: Connecting Algebra and Geometry Through Coordinates

Congruence   G-CO

G.CO.8   Use and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid and kite.

G.CO.9   Characterize parallelograms in terms of equality of opposite sides, in terms of equality of opposite angles, and in terms of bisection of diagonals; characterize rectangles as parallelograms with equal diagonals.

G.CO.12   Use two-dimensional representations to transform figures and to predict the effect of translations, rotations and reflections.

Expressing Geometric Properties with Equations   G-GPE

G.GPE.1   Understand that two lines with well-defined slopes are perpendicular if and only if the product of their slopes is equal to -1.

G.GPE.7   Use the slope criteria for parallel and perpendicular lines to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G.GPE.9   Use coordinates to compute perimeters of polygons and areas for triangles and rectangles, e.g. using the distance formula.★

G.GPE.11   Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.

In this course, focus on theorems involving lengths of segments, polygons, and other situations involving linear relationships. Avoid theorems about circles.

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Pathway B
Course 2

The course is conceived in five units: (1) Number systems, (2) Quadratics and beyond, (3) Applications of probability, (4) Similarity and right triangle trigonometry, and (5) Circles with and without coordinates.

Number Systems. In this unit, students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system. In the real numbers, students explore distinctions between rational and irrational numbers by considering their decimal representations. Based on experiences converting fractions to repeating decimals and converting repeating decimals to fractions, students can reason that irrational numbers have decimal representations that neither repeat nor terminate. These irrational numbers "complete" the real number line. The rules of exponents are extended to include rational exponents, which are related to roots and radicals, and which allow for additional precision in the analysis of exponential functions and equations. The unit culminates with consideration of the equation $x^2 + 1 = 0$, which has no solution on the real number line, but which has a solution in a larger number system called the complex numbers.

Quadratics and Beyond. In this unit, students study quadratic expressions, equations, and functions with real coefficients. Quadratic functions are compared to other functions, including exponential functions and simple cubic functions. Students use area and volume models to interpret polynomial multiplication, as appropriate. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students find complex solutions, and they interpret the existence of non-real solutions as indicating that the graph of the related quadratic function does not cross the horizontal axis. They discuss the fundamental theorem of algebra in order to complete the thread begun in Number Systems unit.

Applications of Probability. Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability.

Similarity and Right Triangle Trigonometry. Building on similarity ideas that were introduced in grade 8 as applications of proportional reasoning, students formalize the study of similarity and identify criteria for similarity of triangles. Similarity in right triangles is formalized as right triangle trigonometry, with special attention to special right triangles and the use of the Pythagorean theorem.

Circles With and Without Coordinates. In this unit, students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In coordinate systems, students use the distance formula to write the equation of a circle. Given an equation of a circle, they draw the graph in the coordinate plane, and they find intersections between a line and a circle and between two circles as an application of techniques for solving quadratic equations.

Unit 1: Number Systems

The Real Number System   N-RN

N.RN.1	Understand that the laws of exponents for positive integer exponents follow from an understanding of exponents as indicating repeated multiplication, and from the associative law for multiplication.
	In this course, focus on integral exponents.
N.RN.2	Understand that the definition of the meaning of zero, positive rational, and negative exponents follows from extending the laws of exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, since $(5^{1/3})^3 = 5^{(1/3)\cdot3} = 5^1 = 5$, $5^{1/3}$ is a cube root of 5.*

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While integral exponents were addressed in the previous course, in this course rational exponents should also be addressed. Be prepared to informally discuss the graph of an exponential function as a continuous curve.

N.RN.3　　Understand that sums and products of rational numbers are rational.

N.RN.4　　Understand that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational

Include definition of rational numbers as infinite and periodic decimals and definition of irrational numbers as infinite but non-periodic decimals.

N.RN.5　　Rewrite expressions using the laws of exponents. *For example $(5^{1/2})^3 = 5^{3/2}$ and $1/5 = 5^{-1}$.*

Seeing Structure in Expressions　　　　　　　　　　　　　　　　　　　A-SSE

A.SSE.6　　Rewrite expressions using the laws of exponents. For example, $(x^{1/2})^3 = x^{3/2}$ and $1/x = x^{-1}$.

Unit 2: Quadratics and Beyond

The Complex Number System　　　　　　　　　　　　　　　　　　　　N-CN

N.CN.1　　Understand that the relation $i^2 = -1$ and the commutative, associative, and distributive laws can be used to calculate with complex numbers.

Include discussion of the extension of the number system to complex numbers based on need to solve certain equations.

N.CN.2　　STEM Understand that polynomials can be factored over the complex numbers, e.g., as in $x^2 + 4 = (x + 2i)(x - 2i)$.

Extend to higher degree polynomials where factoring is reasonable. Include application of factoring polynomials where factoring is reasonable. Include application of factoring polynomials to reduce rational expressions to lowest terms.

N.CN.6　　Add, subtract, and multiply complex numbers.

Include application to quadratics and extend to include cubics and higher powers of i.

N.CN.8　　STEM Solve quadratic equations with real coefficients that have complex solutions.

Seeing Structure in Expressions　　　　　　　　　　　　　　　　　　　A-SSE

A.SSE.1　　Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression *1.15t* can be rewritten as *$(1.15^{1/12})^{12t} \approx 1.012^{12t}$* to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Extend to quadratic, exponential, and simple rational functions in this course.

A.SSE.2　　Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.

A.SSE.4　　Factor, expand, and complete the square in quadratic expressions.

A.SSE.5　　See expressions in different ways that suggest ways of transforming them. For example, see *$x^4 - y^4$ as $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as *$(x^2 - y^2)(x^2 + y^2)$*.

A.SSE.6　　Rewrite expressions using the laws of exponents. For example, *$(x^{1/2})^3 = x^{3/2}$ and $1/x = x^{-1}$.*

In this course, expand the applications of the laws of exponents from the integral　　exponents treated in Course 1 to fractional exponents.

A.SSE.7　　Use the laws of exponents to interpret expressions for exponential functions, recognizing positive rational exponents as indicating roots of the base and negative exponents as indicating the reciprocal of a power. For example, identify the per unit percentage change in functions such as *$y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$,* and conclude whether it represents exponential growth or decay. Recognize that any non-zero number raised to the 0 power is 1 (for example, *$12(1.05)^0 = 12$*). Avoid common errors such as confusing *$6(1.05)^t$ with $(6 \cdot 1.05)^t$ and $5(0.03)^t$ with $5(1.03)^t$.*

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Arithmetic with Polynomials and Rational Expressions A-APR

A.APR.6   Add, subtract and multiply polynomials.

Emphasize relationship of polynomial multiplication as an application of the distributive property. In this course, focus on rewriting polynomial expressions that result in linear or quadratic polynomials or to expressions or equations that can be seen as having fundamentally a linear or quadratic form.

A.APR.7   Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the polynomial.

Focus on quadratic polynomials in this course.

Creating Equations That Describe Numbers or Relationships A-CED★

A.CED.2   Understand that equations in two or more variables that represent a relationship between quantities can be built by experimenting with specific numbers in the relationship.

In this course, focus on examples drawn from quadratic and simple rational relationships such as contexts that exhibit inverse proportionality.

A.CED.3   Write equations and inequalities that specify an unknown quantity or to express a relationship between two or more quantities. Use the equations and inequalities to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Include equations arising from situations involving linear, quadratic, simple rational and exponential functions. Focus on quadratic, simple rational and exponential functions in this course.

A.CED.4   Rearrange formulas to highlight a quantity of interest. *For example, transform Ohm's law $V = IR$ to highlight resistance R; in motion with constant acceleration, transform $v_{f,x}^2 - v_{i,x}^2 = 2a_x(x_f - x_i)$ to highlight the change in position along the x-axis, $x_f - x_i$.*

Focus should be on formulas involving quadratic or rational expressions in this course.

Reasoning with Equations and Inequalities A-REI

A.REI.2   Understand that the method of completing the square can transform any quadratic equation in x into an equivalent equation of the form $(x - p)^2 = q$. This leads to the quadratic formula.

A.REI.5   Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Expand from linear equations addressed in Course 1 to include the graphs of quadratic, exponential and simple rational equations in this course.

A.REI.12   Solve quadratic equations in one variable. *Include methods such as inspection (e.g. for $x^2 = 49$), square roots, completing the square, the quadratic formula and factoring. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*

A.REI.13   Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions.

Include the use of technology to approximate intersections.

A.REI.16   Solve algebraically a simple system consisting of one linear equation and one quadratic equation in two variables; for example, find points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

A.REI.19   In the context of exponential models, solve equations of the form $a\,b^{ct} = d$ where a, c, and d are specific numbers and the base b is 2, 10, or e.★

A.REI.20   STEM Relate the properties of logarithms to the laws of exponents and solve equations involving exponential functions.

Relate to A.SSE.d, the laws of exponents.

Interpreting Functions A-IF

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A.IF.2	Understand that [quadratic] functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.
	In this course, focus on quadratic and simple rational functions but include examples from other types of functions, such as power functions. Compare and contrast with linear and exponential functions.
A.IF.3	Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function.
	In this course, focus especially on different forms of a quadratic function and the properties highlighted in each form.
A.IF.5	Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic).[*]
	In this course, extend beyond linear and exponential relationships to include those that are quadratic or inversely proportional.
A.IF.6	Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities.[*]
	In this course, extend beyond linear and exponential relationships to include those that are quadratic or inversely proportional.
A.IF.9	Describe the qualitative behavior of functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.[*]
	Focus on quadratic and simple rational functions; compare to linear and exponential functions. Do not address trigonometric functions in this course.
A.IF.10	Use technology to exhibit the effects of parameter changes on the graphs of linear, power, quadratic, square root, cube root, and polynomial functions, and simple rational, exponential, logarithmic, sine, cosine, absolute value and step functions.[*]
A.IF.12	Transform quadratic polynomials algebraically to reveal different features of the function they define, such as zeros, extreme values, and symmetry of the graph.

Building Functions A-BF

A.BF.5	Write a function that describes a relationship between two quantities, for example by varying parameters in and combining standard function types (such as linear, quadratic or exponential functions). Experiment with parameters and illustrate an explanation of the behavior of the function when parameters vary using technology.[*]
	Focus on quadratic and simple rational functions in this course.
A.BF.6	Solve problems involving linear, quadratic, and exponential functions.[*]
A.BF.7	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
	Focus on absolute value, quadratic, and simple rational functions in this course.

Linear, Quadratic, and Exponential Models A-LQE

A.LQE.2	Understand that quadratic functions have maximum or minimum values and can be used to model problems with optimum solutions.[*]
A.LQE.6	Understand that in a sequence that increases quadratically (e.g., $a_n = 3n^2 + 2n + 1$), differences between consecutive terms form an arithmetic sequence, and second differences form a constant sequence. Conversely, if the second differences form a constant sequence with non-zero value, the sequence increases quadratically.[*]

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[*] Standard with close connection to modeling

[*] Standard with close connection to modeling

A.LQE.8   Understand that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically or (more generally) as a polynomial function.★

In this course, compare exponential growth to quadratic growth. Note that it is very difficult to distinguish between quadratic and exponential growth over a limited domain.

A.LQE.11   Use quadratic functions to model problems, e.g. in situations with optimum solutions.★

Unit 3: Applications of Probability

Independently Combined Probability Models   S-IPM

S.IPM.1   Understand that to describe a pair of random processes (such as tossing a coin and rolling a number cube), or one random process repeated twice (such as randomly selecting a student in the class on two different days), two probability models can be combined into a single model.

- The sample space for the combined model is formed by listing all possible ordered pairs that combine an individual outcome from the first model with an individual outcome from the second. Each ordered pair is an individual outcome in the combined model.

- The total number of individual outcomes (ordered pairs) in the combined model is the product of the number of individual outcomes in each of the two original models.

S.IPM.2   Understand that when two probability models are combined independently, the probability that one type of outcome in the first model occurs together with another type of outcome in the second model is the product of the two corresponding probabilities in the original models (the Multiplication Rule).

S.IPM.3   Combine two uniform models independently to compute probabilities for a pair of random processes (e.g., flipping a coin twice, selecting one person from each of two classes).

- Use organized lists, tables and tree diagrams to represent the combined sample space.

- Determine probabilities of ordered pairs in the combined model, and determine the probability of a particular type or category of outcomes in the combined model, as the fraction of ordered pairs corresponding to it.

S.IPM.4   For two independently combined uniform models, use the Multiplication Rule to determine probabilities.

Conditional Probability and the Laws of Probability   S-CP

S.CP.1   Understand that events are subsets of a sample space; often, events of interest are defined by using characteristics (or categories) of the sample points, or as unions, intersections, or complements thereof ("and," "or," "not"). A sample point may belong to several events (categories).

S.CP.2   Understand that if A and B are two events, then in a uniform model the conditional probability of A given B, denoted by $P(A \mid B)$, is the fraction of B's sample points that also lie in A.

S.CP.3   Understand that the laws of probability allow one to use known probabilities to determine other probabilities of interest.

S.CP.4   Compute probabilities by constructing and analyzing sample spaces, representing them by tree diagrams, systematic lists, and Venn diagrams.

S.C.5   Use the laws of probability to compute probabilities.

S.CP.6   Apply concepts such as intersections, unions and complements of events, and conditional probability and independence to define or analyze events, calculate probabilities and solve problems.

S.CP.7   Construct and interpret two-way tables to show probabilities when two characteristics (or categories) are associated with each sample point. Use a two-way table to determine conditional probabilities.★

S.CP.8   Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.★

S.CP.9   Use permutations and combinations to compute probabilities of compound events and solve problems.

Experimenting and Simulating to Model Probabilities   S-ES

S.ES.1   Understand that sets of data obtained from surveys, simulations or other means can be used as probability models, by treating the data set itself as a sample space, in which the sample points are the individual pieces of data.

S.ES.2   Understand that the probability of an outcome can be interpreted as an assertion about the long-run proportion of the outcome's occurrence if the random experiment is repeated a large number of times.

S.ES.3   Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.

S.ES.4   Compare the results of simulations with predicted probabilities. When there are substantial discrepancies between predicted and observed probabilities, explain them.

S.ES.5   Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.

Using Probability to Make Decisions   S-MD

S.MD.1   Understand that the expected value of a random variable is the weighted average of its possible values, with weights given by their respective probabilities.

S.MD.2   Understand that when the possible outcomes of a decision can be assigned probabilities and payoff values, the decision can be analyzed as a random variable with an expected value, e.g. of an investment.

S.MD.3   Calculate expected value, e.g. to determine the fair price of an investment.

S.MD.4   Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S.MD.5   Evaluate and compare two investments or strategies with the same expected value, where one investment or strategy is safer than the other.

S.MD.6   Evaluate and compare two investments or strategies, where one investment or strategy is safer but has lower expected value. Include large and small investments, and situations with serious consequences.

S.MD.7   Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).

Unit 4: Similarity, Right Triangle Trigonometry, and Measurement

Similarity, Right Triangles, and Trigonometry   G-SRT

G.SRT.1   Understand that dilating a line produces a line parallel to the original. (In particular, lines passing through the center of the dilation remain unchanged.)

G.SRT.2   Understand that the dilation of a given segment is parallel to the given segment and longer or shorter in the ratio given by the scale factor. A dilation leaves a segment unchanged if and only if the scale factor is 1.

G.SRT.3   Understand that the assumed properties of dilations can be used to establish the AA, SAS, and SSS criteria for similarity of triangles.

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G.SRT.4	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of sine, cosine and tangent.
G.SRT.5	Understand that a line parallel to one side of a triangle divides the other two proportionally, and conversely.
G.SRT.6	Use triangle similarity criteria to solve problems and to prove relationships in geometric figures. Include a proof of the Pythagorean Theorem using triangle similarity.
G.SRT.7	Use and explain the relationship between the sine and cosine of complementary angles.
G.SRT.8	Use sine, cosine, tangent and the Pythagorean Theorem to solve right triangles[§§] in applied problems.
G.SRT.9	STEM Give an informal explanation using successive approximation that a dilation of scale factor r changes the length of a curve by a factor of r and the area of a region by a factor of r^2.

Circles   G-C

| G.C.1 | Understand that dilations can be used to show that all circles are similar. |

Expressing Geometric Properties with Equations   G-GPE

| G.GPE.8 | Find the point on the segment between two given points that divides the segment in a given ratio. |

Modeling with Geometry   G-MG

| G.MG.1 | Understand that models of objects and structures can be built from a library of standard shapes; a single kind of shape can model seemingly different objects.★ |
| G.MG.2 | Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso or as a cylinder).★ |

Unit 5: Circles With and Without Coordinates

Circles   G-C

G.C.1	Understand that dilations can be used to show that all circles are similar.
G.C.2	Understand that here is a unique circle through three non-collinear points, or tangent to three non-concurrent lines.
	Connect to the construction of a circle.
G.C.3	Identify and define radius, diameter, chord, tangent, secant and circumference.
G.C.4	Identify and describe relationships among angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
G.C.5	Determine the arc lengths and the areas of sectors of circles, using proportions.
G.C.6	STEM Construct a tangent line from a point outside a given circle to the circle.
G.C.7	STEM Prove and use theorems about circles, and use these theorems to solve problems. *Include:*

- *Symmetries of a circle*

- *Similarity of a circle to any other*

- *Tangent line, perpendicularity to a radius*

- *Inscribed angles in a circle, relationship to central angles, and equality of inscribed angles*

[§§] A right triangle has five parameters, its three lengths and two acute angles. Given a length and any other parameter, "solving a right triangle" means finding the remaining three parameters.

★ Standard with close connection to modeling

- *Properties of chords, tangents and secants as an application of triangle similarity.*

Expressing Geometric Properties with Equations G-GPE

G.GPE.2   Understand that the equation of a circle can be found using its definition and the Pythagorean Theorem.

G.GPE.3   Understand that transforming the graph of an equation by reflecting in the axes, translating parallel to the axes, or applying a dilation to one of the axes correspond to substitutions in the equation. *For example, reflection in the y axis corresponds to (x,y) → (−x,y), translation vertically down by three units corresponds to (x,y) → (x,y+3), and dilating by a factor of 2 parallel to the x-axis corresponds to (x, y) → (x/2, y).*

G.GPE.10   Decide whether a point with given coordinates lies on a circle defined by a given equation.

G.GPE.11   Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).

G.GPE.12   Complete the square to find the center and radius of a circle given by an equation.

Include the case where the radius is 1 with center at the origin, and an introductory discussion of the unit circle.

Modeling with Geometry G-MG

G.MG.1   Understand that models of objects and structures can be built from a library of standard shapes; a single kind of shape can model seemingly different objects.[★]

G.MG.2   Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso or as a cylinder).[★]

[★] Standard with close connection to modeling

Appendix A
Designing High School Mathematics Courses Based on the Common Core Standards

Pathway B
Course 3a

During the first semester of Course 3 in the Integrated Mathematics sequence students engage in coherent study of beginning statistics. They see how the visual displays and summary statistics they learned in earlier grades apply to different types of data and to probability distributions of those data. They identify different ways of collecting data – including sample surveys, experiments, and simulations – and the role that randomness and careful design play in the conclusions that can be drawn. Students learn and apply the laws of probability and begin to understand the fundamental role of probability in statistical processes.

With patterns of data as a foundation, students are prepared to use their increasing repertoire of equations and functions to model a wide variety of contextual situations mathematically. The second semester of Course 3 adds power, polynomial, additional rational functions to the linear, exponential, and quadratic functions of Course 2. Qualitative analysis of these functions shows students how to adjust a function to better model a particular problem situation or set of data. Students understand that the effect of certain function transformations is uniform across function types and begins to understand the power of generalization in mathematics. They understand the analogy between the system of integers and the system of polynomials. They perform operations on complex numbers, seeing this set of numbers as an expansion of the real number system. The generalization theme continues in geometry where the law of sines and the law of cosines extend the application of trigonometry to non-right triangles.

Unit 1: Inferences and Conclusions from Data

Making Inferences and Justifying Conclusions   S-IC

S.IC.1	Understand that statistics is a process for making inferences about population parameters based on a sample from that population; randomness is the foundation for statistical inference.
S.IC.2	Understand that the design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.
S.IC.3	Understand that simulation-based techniques are powerful tools for making inferences and justifying conclusions from data.
S.IC.4	Use probabilistic reasoning to decide if a specified model is consistent with results from a given data-generating process. (For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?)
S.IC.5	Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each.
S.IC.6	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S.IC.7	Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment.
S.IC.8	Evaluate reports based on data.

Experimenting and Simulating to Model Probabilities   S-ES

S.ES.1	Understand that sets of data obtained from surveys, simulations or other means can be used as probability models, by treating the data set itself as a sample space, in which the sample points are the individual pieces of data.
S.ES.5	Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.

Include using the normal distribution as an approximation to the binomial distribution and connect this with the Binomial Theorem, Pascal's Triangle and Probability.

Unit 2: Polynomial Expressions and Functions

The Complex Number System N-CN

N.CN.2   STEM Understand that polynomials can be factored over the complex numbers, e.g., as in $x^2 + 4 = (x + 2i)(x − 2i)$.

This was introduced in Course 2 for quadratics. In this course, it should be extended to higher degree polynomials where factoring is reasonable. Include application of factoring polynomials where factoring is reasonable. Include application of factoring polynomials to reduce rational expressions to lowest terms.

N.CN.6   Add, subtract, and multiply complex numbers.

Extend beyond the application to quadratics and $i^2 = −1$ to include cubics and higher powers of i.

Seeing Structure in Expressions A-SSE

A.SSE.1   Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} ≈ 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.SSE.2   Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.SSE.5   See expressions in different ways that suggest ways of transforming them. For example, see $x^4 − y^4$ as $(x^2)^2 − (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 − y^2)(x^2 + y^2)$.

In this course, focus on polynomial, rational, square root and cube root expressions

Arithmetic with Polynomials and Rational Expressions A-APR

A.APR.1   Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.

A.APR.2   Understand that polynomial identities become true statements no matter which real numbers are substituted. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 − y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

Extend beyond quadratics addressed in Course 2 to include higher degree polynomials in this course.

A.APR.3   Understand the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x − a$ is $p(a)$, so $p(a) = 0$ if and only if $(x − a)$ is a factor of $p(x)$.

Relate long division of polynomials to long division of positive integers.

A.APR.4   STEM Understand that the Binomial Theorem gives the expansion of $(x + a)^n$ in powers of x for a positive integer n and a real number a, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Connect to probability and Pascal's triangle. The proof may be explained by the instructor, but is not intended to be performed by students on their own.

A.APR.6   Add, subtract and multiply polynomials.

Build on experiences from Course 2 with polynomials that resulted in linear or quadratic expressions to include operations that result in polynomials of a higher degree.

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A.APR.7	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the polynomial.
	Extend work with quadratics to higher degree polynomials when suitable factorizations are available.

Reasoning with Equations and Inequalities A-REI

A.REI.4	Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane, often forming a curve or a line.
	In this course, focus on polynomial, rational, square root and cube root expressions.
A.REI.5	Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
	In this course, focus on polynomial, rational, square root and cube root expressions.
A.REI.13	Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions.
	In this course, expand the type of functions to include polynomial, rational, square root, and cube root functions in this course. Include the use of technology to approximate intersections.

Interpreting Functions A-IF

A.IF.2	Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.
	Focus on polynomial, rational, square root and cube root functions in this course. Note the information given by factored forms of each of these types of functions.
A.IF.3	Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function.
	Focus on rational, square root, and cube root functions in this course. Note the information given by factored forms of each of these types of functions, especially related to roots and asymptotes.
A.IF.5	Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic). ★
	Extend to polynomial, rational, square root, and cube root functions in this course.
A.IF.6	Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities.★
	Extend to polynomial, rational, square root, and cube root functions in this course.
A.IF.10	Use technology to exhibit the effects of parameter changes on the graphs of linear, power, quadratic, square root, cube root, and polynomial functions, and simple rational, exponential, logarithmic, sine, cosine, absolute value and step functions.★
	While the focus of this standard is on the effects of parameter changes on the graphs of several functions, it is advisable to consider introducing the graphs through brief examinations of the functions' domains.
	For logarithmic, absolute value and step functions, demonstrate the genesis of their graphs by evaluating the functions for several inputs in their domains and graphing the results.
	While the trigonometric functions will be fully treated in the Precalculus course, one may describe the genesis of their graphs for the purpose of studying transformations. This can be done by demonstrating the use of benchmark acute angles with vertex at the origin and right angles formed with the x-axis and points on the unit circle to develop a table of values in the first quadrant. Demonstrate the use transformations to move the angles to subsequent quadrants eventually generalizing to all quadrants to continue the table. Use the tables to generate the graph of sine and cosine.

★ Standard with close connection to modeling

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A.BF.7   Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k
             (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an
             explanation of the effects on the graph using technology.

             *Apply to polynomial, rational, square root, and cube root functions in this course and consider examples from other types of
             functions. Note the common effect of each of these transformations across function types.*

Unit 3: Simple[9] Rational Expressions and Functions

A.SSE.1   Understand that different forms of an expression may reveal different properties of the quantity in
             question; a purpose in transforming expressions is to find those properties. Examples: factoring a quadratic
             expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or
             minimum value; the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly
             interest rate if the annual rate is 15%.

             In this course, focus on polynomial, rational, square root and cube root expressions.

A.SSE.2   Understand that complicated expressions can be interpreted by viewing one or more of their parts as single
             entities.

             In this course, focus on polynomial, rational, square root and cube root expressions

A.SSE.5   See expressions in different ways that suggest ways of transforming them. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus
             recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

             In this course, focus on polynomial, rational, square root and cube root expressions

A.APR.5   STEM Understand that rational expressions are quotients of polynomials. They form a system analogous to the
             rational numbers, closed under division by a nonzero rational function.

A.APR.8   Transform simple rational expressions using the commutative, associative, and distributive laws, and the inverse
             relationship between multiplication and division.

             *Relate to division by zero. Clarify this it is the function output value that becomes zero with polynomial division. Distinguish
             between the output variable and the input value of the function that makes the function equal zero.*

A.APR.9   Divide a polynomial $p(x)$ by a divisor of the form $x - a$ using long division.

A.APR.11   STEM Identify zeros and asymptotes of rational functions, when suitable factorizations are available, and use the
             zeros and asymptotes to construct a rough graph of the function.

A.APR.12   STEM Divide polynomials, using long division for linear divisors and long division or a computer algebra system for
             higher degree divisors.

A.REI.4   Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane,
             often forming a curve or a line.

             In this course, focus on polynomial, rational, square root and cube root expressions.

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[9] In this course "simple rational" expressions are taken to be those that are at most linear polynomials divided by quadratic polynomials.

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A.REI.5   Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.REI.10   Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions.

Focus on rational, square root and cube root[10] functions.

A.REI.13   Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions.

In this course, expand the type of functions to include polynomial, rational, square root and cube root expressions. Include the use of technology to approximate intersections.

Interpreting Functions   A-IF

A.IF.2   Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.

Focus on polynomial, rational, square root and cube root functions in this course.

A.IF.3   Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function

Focus on polynomial, rational, square root, and cube root functions in this course. Note the information given by factored forms of each of these types of functions, especially related to roots and asymptotes.

A.IF.5   Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic). ★

Extend to polynomial, rational, square root, and cube root functions in this course.

A.IF.6   Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities.★

Extend to polynomial, rational, square root, and cube root functions in this course.

A.IF.9   Describe the qualitative behavior of functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★

Extend to polynomial, rational, square root, and cube root functions in this course.

A.IF.10   Use technology to exhibit the effects of parameter changes on the graphs of linear, power, quadratic, square root, cube root, and polynomial functions, and simple rational, exponential, logarithmic, sine, cosine, absolute value and step functions.★

While the focus of this standard is on the effects of parameter changes on the graphs of several functions, it is advisable to consider introducing the graphs through brief examinations of the functions' domains.

For logarithmic, absolute value and step functions, demonstrate the genesis of their graphs by evaluating the functions for several inputs in their domains and graphing the results.

While the trigonometric functions will be fully treated in the Precalculus course, one may describe the genesis of their graphs for the purpose of studying transformations. This can be done by demonstrating the use of benchmark acute angles with vertex at the origin and right angles formed with the x-axis and points on the unit circle to develop a table of values in the first

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[10] The introduction of the square root and cube root functions should be handled with care. In particular, their problematic nature over the complex numbers should be discussed as existing and to be encountered fully in later mathematics courses.

★ Standard with close connection to modeling

quadrant. Demonstrate the use transformations to move the angles to subsequent quadrants eventually generalizing to all quadrants to continue the table. Use the tables to generate the graph of sine and cosine.

Building Functions A-BF

| A.BF.7 | Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. |

Apply to polynomial, rational, square root, and cube root functions in this course and consider examples from other types of functions. Note the common effect of each of these transformations across function types.

Unit 4: Modeling with Functions

Seeing Structure in Expressions A-SSE

| A.SSE.3 | Interpret an expression that represents a quantity in terms of the context. Include interpreting parts of an expression, such as terms, factors and coefficients. ★ |

Reasoning with Equations and Inequalities A-REI

| A.REI.10 | Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions. |
| A.REI.18 | In modeling situations, represent constraints by systems of equations and/or inequalities, and interpret solutions of these systems as viable or non-viable options in the modeling context. ★ |

In this course, expand the type of functions to include polynomial, rational, square root and cube root equations or inequalities.

Interpreting Functions A-IF

| A.IF.2 | Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior. |

Focus on polynomial, rational, square root and cube root functions in this course.

| A.IF.5 | Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic). ★ |

Extend to polynomial, rational, square root, and cube root functions in this course.

| A.IF.6 | Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities. ★ |

Extend to polynomial, rational, square root, and cube root functions in this course.

| A.IF.8 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★ |

Extend to polynomial, rational, square root, and cube root functions in this course.

| A.IF.9 | Describe the qualitative behavior of functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★ |

Extend to polynomial, rational, square root, and cube root functions in this course.

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A.BF.7	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
	Apply to polynomial, rational, square root, and cube root functions in this course and consider examples from other types of functions. Note the common effect of each of these transformations across function types.

Unit 5: Geometry of General Triangles

G.TGT.1	STEM Understand that the formula A = ½ ab sin(C) for the area of a triangle can be derived by drawing an auxiliary line from a vertex perpendicular to the opposite side. Applying this formula in three different ways leads to the Law of Sines.
G.TGT.2	STEM Understand that the Law of Cosines generalizes the Pythagorean Theorem.
G.TGT.4	STEM Understand that the Laws of Sines and Cosines embody the triangle congruence criteria, in that three pieces of information are usually sufficient to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that "Side-Side-Angle" is not a congruence criterion.
	Do not emphasize ambiguous case in this course. Constrain to a discussion of its existence.
G.TGT.5	STEM Explain proofs of the Law of Sines and the Law of Cosines.
	For proof, emphasize explanation.
G.TGT.6	STEM Use the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
	Do not include ambiguous case.

G.GMD.1	Understand that the area of a decomposed figure is the sum of the areas of its components and is independent of the choice of dissection.
G.GMD.4	Find areas of polygons by dissecting them into triangles.
G.GMD.5	Explain why the volume of a cylinder is the area of the base times the height, using informal arguments.
G.GMD.6	For a pyramid or a cone, give a heuristic argument to show why its volume is one-third of its height times the area of its base.
G.GMD.7	Apply formulas and solve problems involving volume and surface area of right prisms, right circular cylinders, right pyramids, cones, spheres and composite figures.

G.MG.3	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★
G.MG.4	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy constraints or minimize cost; working with typograph.

★ Standard with close connection to modeling

Appendix A
Designing High School Mathematics Courses Based on the Common Core Standards

Pathway B
Course 3b

During the first semester of Course 3 in the Integrated Mathematics sequence students engage in coherent study of beginning statistics. They see how the visual displays and summary statistics they learned in earlier grades apply to different types of data and to probability distributions of those data. They identify different ways of collecting data – including sample surveys, experiments, and simulations – and the role that randomness and careful design play in the conclusions that can be drawn. Students learn and apply the laws of probability and begin to understand the fundamental role of probability in statistical processes.

With patterns of data as a foundation, students are prepared to use their increasing repertoire of equations and functions to model a wide variety of contextual situations mathematically. The second semester of Course 3 adds power, polynomial, additional rational functions to the linear, exponential, and quadratic functions of Course 2. Qualitative analysis of these functions shows students how to adjust a function to better model a particular problem situation or set of data. Students understand that the effect of certain function transformations is uniform across function types and begins to understand the power of generalization in mathematics. They understand the analogy between the system of integers and the system of polynomials. The generalization theme continues in geometry where the law of sines and the law of cosines extend the application of trigonometry to non-right triangles.

Additional emphasis is placed in this course on modeling. Students should engage with meaningful and rigorous modeling tasks to deepen their understanding of the mathematics.

Unit 1: Inferences and Conclusions from Data

Making Inferences and Justifying Conclusions   S-IC

S.IC.1   Understand that statistics is a process for making inferences about population parameters based on a sample from that population; randomness is the foundation for statistical inference.

S.IC.2   Understand that the design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.

S.IC.3   Understand that simulation-based techniques are powerful tools for making inferences and justifying conclusions from data.

S.IC.4   Use probabilistic reasoning to decide if a specified model is consistent with results from a given data-generating process. (For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?)

S.IC.5   Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each.

S.IC.6   Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.7   Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment.

S.IC.8   Evaluate reports based on data.

Experimenting and Simulating to Model Probabilities   S-ES

S.ES.1   Understand that sets of data obtained from surveys, simulations or other means can be used as probability models, by treating the data set itself as a sample space, in which the sample points are the individual pieces of data.

S.ES.5	Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.

Include using the normal distribution as an approximation to the binomial distribution and connect this with the Binomial Theorem, Pascal's Triangle and Probability.

Unit 2: Polynomial Expressions and Functions

Seeing Structure in Expressions A-SSE

A.SSE.1	Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.SSE.2	Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.SSE.5	See expressions in different ways that suggest ways of transforming them. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

In this course, focus on polynomial, rational, square root and cube root expressions

Arithmetic with Polynomials and Rational Expressions A-APR

A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.
A.APR.2	Understand that polynomial identities become true statements no matter which real numbers are substituted. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Extend beyond quadratics addressed in Course 2 to include higher degree polynomials in this course.

A.APR.3	Understand the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Relate long division of polynomials to long division of positive integers.

A.APR.6	Add, subtract and multiply polynomials.

Build on experiences from Course 2 with polynomials that resulted in linear or quadratic expressions to include operations that result in polynomials of a higher degree.

A.APR.7	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the polynomial.

Extend work with quadratics to higher degree polynomials when suitable factorizations are available.

Reasoning with Equations and Inequalities A-REI

A.REI.4	Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane, often forming a curve or a line.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.REI.5	Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

In this course, focus on polynomial, rational, square root and cube root expressions.

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A.REI.13   Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions.

In this course, expand the type of functions to include polynomial, rational, square root, and cube root functions in this course. Include the use of technology to approximate intersections.

Interpreting Functions A-IF

A.IF.2   Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.

Focus on polynomial, rational, square root and cube root functions in this course. Note the information given by factored forms of each of these types of functions.

A.IF.3   Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function.

Focus on rational, square root, and cube root functions in this course. Note the information given by factored forms of each of these types of functions, especially related to roots and asymptotes.

A.IF.5   Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic). ★

Extend to polynomial, rational, square root, and cube root functions in this course.

A.IF.6   Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities.★

Extend to polynomial, rational, square root, and cube root functions in this course.

A.IF.10   Use technology to exhibit the effects of parameter changes on the graphs of linear, power, quadratic, square root, cube root, and polynomial functions, and simple rational, exponential, logarithmic, sine, cosine, absolute value and step functions.★
While the focus of this standard is on the effects of parameter changes on the graphs of several functions, it is advisable to consider introducing the graphs through brief examinations of the functions' domains.

For logarithmic, absolute value and step functions, demonstrate the genesis of their graphs by evaluating the functions for several inputs in their domains and graphing the results.

While the trigonometric functions will be fully treated in the Precalculus course, one may describe the genesis of their graphs for the purpose of studying transformations. This can be done by demonstrating the use of benchmark acute angles with vertex at the origin and right angles formed with the x-axis and points on the unit circle to develop a table of values in the first quadrant. Demonstrate the use transformations to move the angles to subsequent quadrants eventually generalizing to all quadrants to continue the table. Use the tables to generate the graph of sine and cosine.

Building Functions A-BF

A.BF.7   Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Apply to polynomial, rational, square root, and cube root functions in this course and consider examples from other types of functions. Note the common effect of each of these transformations across function types.

★ Standard with close connection to modeling

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Unit 3: Simple[11] Rational Expressions and Functions

Seeing Structure in Expressions
A-SSE

A.SSE.1   Understand that different forms of an expression may reveal different properties of the quantity in question; a purpose in transforming expressions is to find those properties. Examples: factoring a quadratic expression reveals the zeros of the function it defines, and putting the expression in vertex form reveals its maximum or minimum value; the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.SSE.2   Understand that complicated expressions can be interpreted by viewing one or more of their parts as single entities.

In this course, focus on polynomial, rational, square root and cube root expressions

A.SSE.5   See expressions in different ways that suggest ways of transforming them. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

In this course, focus on polynomial, rational, square root and cube root expressions

Arithmetic with Polynomials and Rational Expressions
A-APR

A.APR.5   <small>STEM</small> Understand that rational expressions are quotients of polynomials. They form a system analogous to the rational numbers, closed under division by a nonzero rational function.

A.APR.8   Transform simple rational expressions using the commutative, associative, and distributive laws, and the inverse relationship between multiplication and division.

Relate to division by zero. Clarify this it is the function output value that becomes zero with polynomial division. Distinguish between the output variable and the input value of the function that makes the function equal zero.

A.APR.9   Divide a polynomial $p(x)$ by a divisor of the form $x - a$ using long division.

Reasoning with Equations and Inequalities
A-REI

A.REI.4   Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane, often forming a curve or a line.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.REI.5   Understand that solutions to two equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

In this course, focus on polynomial, rational, square root and cube root expressions.

A.REI.10   Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions.

Focus on rational, square root and cube root[12] functions.

A.REI.13   Solve equations $f(x) = g(x)$ approximately by finding the intersections of the graphs of $f(x)$ and $g(x)$, e.g. using technology to graph the functions. Include cases where f(x) and/or g(x) are linear, polynomial, rational, exponential, and logarithmic functions.

In this course, expand the type of functions to include polynomial, rational, square root and cube root expressions. Include the use of technology to approximate intersections.

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[11] In this course "simple rational" expressions are taken to be those that are at most linear polynomials divided by quadratic polynomials.

[12] The introduction of the square root and cube root functions should be handled with care. In particular, their problematic nature over the complex numbers should be discussed as existing and to be encountered fully in later mathematics courses.

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A.IF.2	Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.
	Focus on polynomial, rational, square root and cube root functions in this course.
A.IF.3	Understand that a function defined by an expression may be written in different but equivalent forms, which can reveal different properties of the function
	Focus on polynomial, rational, square root, and cube root functions in this course. Note the information given by factored forms of each of these types of functions, especially related to roots and asymptotes.
A.IF.5	Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic). ★
	Extend to polynomial, rational, square root, and cube root functions in this course.
A.IF.6	Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities.★
	Extend to polynomial, rational, square root, and cube root functions in this course.
A.IF.9	Describe the qualitative behavior of functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
	Extend to polynomial, rational, square root, and cube root functions in this course.
A.IF.10	Use technology to exhibit the effects of parameter changes on the graphs of linear, power, quadratic, square root, cube root, and polynomial functions, and simple rational, exponential, logarithmic, sine, cosine, absolute value and step functions.★
	While the focus of this standard is on the effects of parameter changes on the graphs of several functions, it is advisable to consider introducing the graphs through brief examinations of the functions' domains.
	For logarithmic, absolute value and step functions, demonstrate the genesis of their graphs by evaluating the functions for several inputs in their domains and graphing the results.
	While the trigonometric functions will be fully treated in the Precalculus course, one may describe the genesis of their graphs for the purpose of studying transformations. This can be done by demonstrating the use of benchmark acute angles with vertex at the origin and right angles formed with the x-axis and points on the unit circle to develop a table of values in the first quadrant. Demonstrate the use transformations to move the angles to subsequent quadrants eventually generalizing to all quadrants to continue the table. Use the tables to generate the graph of sine and cosine.

A.BF.7	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
	Apply to polynomial, rational, square root, and cube root functions in this course and consider examples from other types of functions. Note the common effect of each of these transformations across function types.

★ Standard with close connection to modeling

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Modeling

   Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

   A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.
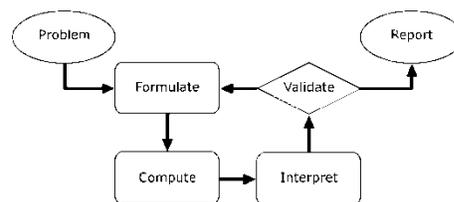
   Some examples of such situations might include:
   - Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
   - Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player
   - Designing the layout of the stalls in a school fair so as to raise as much money as possible.
   - Analyzing stopping distance for a car
   - Modeling savings account balance, bacterial colony growth, or investment growth
   - Critical path analysis, e.g. applied to turnaround of an aircraft at an airport.
   - Risk situations, like extreme sports, pandemics and terrorism.
   - Relating population statistics to individual predictions

   In situation like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

   One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

   The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those than represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then, either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions and approximations are present throughout this cycle.

   In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

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Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, CAS environments, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Unit 4: Modeling with Functions

Seeing Structure in Expressions A-SSE

A.SSE.3   Interpret an expression that represents a quantity in terms of the context. Include interpreting parts of an expression, such as terms, factors and coefficients. ★

Reasoning with Equations and Inequalities A-REI

A.REI.10   Solve simple rational and radical equations in one variable, noting and explaining extraneous solutions.

A.REI.18   In modeling situations, represent constraints by systems of equations and/or inequalities, and interpret solutions of these systems as viable or non-viable options in the modeling context. ★

In this course, expand the type of functions to include polynomial, rational, square root and cube root equations or inequalities.

Interpreting Functions A-IF

A.IF.2   Understand that functions of a single variable have key characteristics, including: zeros; extreme values; average rates of change (over intervals); intervals of increasing, decreasing and/or constant behavior; and end behavior.

Focus on polynomial, rational, square root and cube root functions in this course.

A.IF.5   Describe qualitatively the functional relationship between two quantities by reading a graph (e.g., where the function is increasing or decreasing, what its long-run behavior appears to be, and whether it appears to be periodic). ★

Extend to polynomial, rational, square root, and cube root functions in this course.

A.IF.6   Sketch a graph that exhibits the qualitative features of a function that models a relationship between two quantities. ★

Extend to polynomial, rational, square root, and cube root functions in this course.

A.IF.8   Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★

Extend to polynomial, rational, square root, and cube root functions in this course.

A.IF.9   Describe the qualitative behavior of functions presented in graphs and tables. Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★

Extend to polynomial, rational, square root, and cube root functions in this course.

Building Functions A-BF

A.BF.7   Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

★ Standard with close connection to modeling

Apply to polynomial, rational, square root, and cube root functions in this course and consider examples from other types of functions. Note the common effect of each of these transformations across function types.

Unit 5: Geometry of General Triangles

Trigonometry of General Triangles G-TGT

G.TGT.1   STEM   Understand that the formula A = ½ ab sin(C) for the area of a triangle can be derived by drawing an auxiliary line from a vertex perpendicular to the opposite side. Applying this formula in three different ways leads to the Law of Sines.

G.TGT.2   STEM Understand that the Law of Cosines generalizes the Pythagorean Theorem.

G.TGT.4   STEM Understand that the Laws of Sines and Cosines embody the triangle congruence criteria, in that three pieces of information are usually sufficient to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that "Side-Side-Angle" is not a congruence criterion.

Do not emphasize ambiguous case in this course. Constrain to a discussion of its existence.

G.TGT.5   STEM Explain proofs of the Law of Sines and the Law of Cosines.

For proof, emphasize explanation.

G.TGT.6   STEM Use the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Do not include ambiguous case.

Geometric Measurement and Dimension G-GMD

G.GMD.1   Understand that the area of a decomposed figure is the sum of the areas of its components and is independent of the choice of dissection.

G.GMD.4   Find areas of polygons by dissecting them into triangles.

G.GMD.5   Explain why the volume of a cylinder is the area of the base times the height, using informal arguments.

G.GMD.6   For a pyramid or a cone, give a heuristic argument to show why its volume is one-third of its height times the area of its base.

G.GMD.7   Apply formulas and solve problems involving volume and surface area of right prisms, right circular cylinders, right pyramids, cones, spheres and composite figures.

Modeling with Geometry G-MG

G.MG.3   Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★

G.MG.4   Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy constraints or minimize cost; working with typograph.

★ Standard with close connection to modeling